

ISSN 2310-6697

toiser—open transactions on independent scientific-engineering research

# FUNKTECHNIKPLUS # JOURNAL

Théorie—Expérimentation—Métrologie—Logiciel—Applications

ISSUE 1 – MONDAY 30 SEPTEMBER 2013 – YEAR 1

- 1 Contents
- 2 About
- 3 Editorial Board – Technical Support
- 4 Information for Peers – Guiding Principles
- # Telecommunications Engineering – Applications
- 7 All-Band 2G+3G Radial Disc-Cone Antennas:  
Design, Construction, and Measurements  
N.I. Yannopoulou, P.E. Zimourtopoulos, E.T. Sarris
- # Telecommunications Engineering – Métrologie
- 17 Measurement Uncertainty in Network Analyzers:  
Differential Error Analysis of Error Models Part 1:  
Full One-Port Calibration  
N.I. Yannopoulou, P.E. Zimourtopoulos
- 23 Measurement Uncertainty in Network Analyzers:  
Differential Error Analysis of Error Models Part 2:  
Full Two-Port Calibration  
N.I. Yannopoulou, P.E. Zimourtopoulos



## About

This European Journal defends  
honesty in science and ethics in engineering

**Publisher** – **otoiser** open transactions on independent scientific-engineering research – **ARG NP UoP** Antennas Research Group Non Profit Union of Persons – Hauptstraße 52, 2831 Scheiblingkirchen, Austria – **www.otoiser.org** – **www.antennas.gr**

**Language** – We declare the origins of the Journal by using English, German and French, as well as, a Hellenic vignette, in the cover page. However, since we recognize the dominance of USA English in technical literature, we adopted it as the Journal's language, although it is not our native language.

**Focus** – We consider Radio-FUNK, which still creates a vivid impression of the untouchable, and its Technology-TECHNIK, from an Advanced-PLUS point of view. That is, we dynamically focus at any scientific-engineering discipline producing FUNKTECHNIKPLUS Théorie, Expérimentation, Métrologie, Logiciel, ou Applications.

**Scope** – We emphasize this scope broadness by extending the title of the Journal with a Doppelkreuz-Zeichen #, which we use as a placeholder for the disciplines of Editorial Board:

# Electrical – # Electronics – # Computer – # Telecommunications – Engineering # Computer Science # Applied Mathematics, etc.

**Frequency** – We regularly publish 3 issues per year on January, May, and September as well as an issue every 3 papers.

**Editions** – We follow the arXiv.org system, that is the 1st edition date of every issue is on the cover page and any update date is on the odd pages with its version on all pages.

**Format** – We use the uncommon for Journals A5 paper size and the much readable Liberation Mono fixed-space font, with hyphenation, justification, and unfixed word spacing, to display 2 pages side-by-side on widescreen monitors with WYSIWYG printout. We can consider only papers in ODF odt or MS doc format written in LibreOffice or MS Office with MathType. We use PDFCreator and PDF-Xchange to produce pdf and export to images for printing on demand and electronic publishing.

**License** – We use the free Open Journal System OJS from the PKP Public Knowledge Project for internet publication

**Copyright** – Creative Commons Attribution 3 Unported CC-BY 3

Please download the latest *About* edition from  
[about.ftpj.otoiser.org](http://about.ftpj.otoiser.org)

## **Editorial Board**

### **# Electrical Engineering**

# High Voltage Engineering # Insulating Materials

Professor Michael Danikas

EECE, Democritus University of Thrace, mdanikas@ee.duth.gr

# Electrical Machines and Drives # Renewable Energies

# Electric Vehicles

Assistant Professor Athanasios Karlis

EECE, Democritus University of Thrace, akarlis@ee.duth.gr

### **# Computer Engineering # Software Engineering**

# Cyber Security

Associate Professor Vasileios Katos

EECE, Democritus University of Thrace, vkatos@ee.duth.gr

### **# Internet Engineering # Computer Science**

# Simulation # Applied Education # Learning Management Systems

Lecturer Sotirios Kontogiannis

BA, TE Institute of Western Macedonia, skontog@gmail.com

### **# Applied Mathematics # Functional and Numerical Analysis**

# Applied Electromagnetics # Control Theory

Dr. Nikolaos Berketis, Athens, Greece

BS-Math, MSc-Appl.Math, PhD-Appl.Math, nberketis@gmail.com

### **# Telecommunications Engineering**

# Applied Electromagnetics # Metrology # Applied Education

# Simulation # Virtual Reality # Amateur Radio # FLOSS

# Applied Physics # Electronics Engineering

Dr. Nikolitsa Yannopoulou, Scheiblingkirchen, Austria

Diploma Eng-EE, MEng-EECE, PhD-EECE, yin@arg-at.eu

Assistant Professor Petros Zimourtopoulos

EECE, Democritus University of Thrace, pez@ee.duth.gr

## **Technical Support**

Konstantinos Kondylis, Doha, Qatar

Diploma Eng-EECE, MEng-EECE, k8k@arg-at.eu

Christos Koutsos, Bratislava, Slovakia

Diploma Eng-EECE, MEng-EECE, cak@arg-at.eu

## Information for Peers Guiding Principles

This is a small, but independent, low profile Journal, in which we are all – Authors – Reviewers – Readers – Editors – free at last to be Peers in Knowledge, without suffering from either:

- Journal roles or positions,
- Professional, amateur, or academic statuses, or
- Established impact factorizations,

but with the following Guiding Principles:

**Authors** – We do know what work means; and we do respect the research work of the scientist – engineer; and we do want to highlight this work; and we do decided to found this Journal; and we do publish this work openly; and furthermore, we do care for the work of the technical author, especially the beginner one, whom we do support strongly as follows:

- 1 We encourage the author to submit his own paper written just in Basic English plus Technical Terminology.
- 2 We encourage the author even to select a pen name, which may drop it at any time to reveal his identity.
- 3 We encourage the author to submit an accepted for publication paper, which he was forced to decline that publication because it would be based on a review with unacceptable evaluation or comments.
- 4 We encourage the author to resubmit a non accepted for publication paper that was rejected after a poor, inadequate, unreasonable, irresponsible, incompetent, or ticking only, review.
- 5 We encourage the author to submit a paper that he already self-archived on some open repository, such as arXiv.org.
- 6 We encourage the author to self-archive all of the preprint and postprint versions of his paper.
- 7 We provide the author with a decent, express review process of up to just 4 weeks, by at least 2 peers, never have been co-authors with him.
- 8 We provide the author with a selection of review process between: a traditional, anonymous, peer review decision, and an immediate online pre-publication of his paper followed by an open discussion with at least 2 reviewers.

## About

- 9 We immediately accept for publication a research paper directly resulting from a Project Report, Diploma-, Master-, or PhD-thesis, which the author already has successfully defended before a committee of experts and he can mention 2 members of it, who reviewed and approved his work.
- 10 We immediately publish online a paper, as soon as, it is accepted for publication in the Journal.
- 11 We quickly print-on-demand an issue every 3 papers, in excess of the 3 issues we regularly publish 3 times a year.
- 12 We do not demand from the author to transfer his own copyright to us.
- 13 Nevertheless, we publish only an original research work paper and only if the author does assure us that he owns the copyright of his own paper and submit this paper or a revised version of it to the Journal for publication or even for republication under the Creative Commons–Attribution 3.0 Unported License, CC-BY 3.

**Reviewers** – Any peer may voluntarily become a reviewer of the Journal in his expertise for as long as he wishes. Each author of the Journal has to support the peer-to-peer review process by reviewing one paper, written by non co-author(s) of him, for every one of his papers published in the Journal.

**Readers** – Any reader is a potential reviewer. We welcome online comments and post-reviews by the readers.

**Editors** – Any editor holds a PhD degree, to objectively prove that he really has the working experience of passing through the established publishing system. An editor pre-reviews a paper in order to check its compliance to the Guiding Principles and to select the appropriate reviewers of the Journal. We can only accept for consideration papers in the expertise areas currently shown in the Editorial Board page. However, since we are willing to amplify and extend the Scope of the Journal, we welcome the volunteer expert, in any subject included into it, who wants to join the Board, if he unreservedly accepts our Guiding Principles.

Antennas Research Group – Non-Profit 12(3) \* Union of Persons

\* The Constitution of Greece, Article: 12(3) 2008:

[www.hellenicparliament.gr/en/Vouli-ton-Ellinon/To-Politevma](http://www.hellenicparliament.gr/en/Vouli-ton-Ellinon/To-Politevma)

\* The Hellenic Supreme Court of Civil and Penal Law:

[www.areiospagos.gr/en/](http://www.areiospagos.gr/en/) – Court Rulings:Civil|A1|511|2008

**Submissions**

sub@ftpj.otoiser.org

**Electronic Publishing**

www.ftpj.otoiser.org

**Printing-on-Demand**

pod@ftpj.otoiser.org

Directly:

Georgios Tontrias, msn.expresscopy.xan@hotmail.com  
ExpressCopy, V.Sofias 8, 671 00, Xanthi, Greece

**More** – Detailed information will be available soon at:  
www.ftpj.otoiser.org/gp

# All-Band 2G+3G Radial Disc-Cone Antennas: Design, Construction and Measurements

N.I. Yannopoulou, P.E. Zimourtopoulos, E.T. Sarris \*

Antennas Research Group, Austria – Hellas [1, 2]  
EECE Dept, Democritus University of Thrace, Hellas [2 and 3]

## Abstract

We define as "All-Band 2G+3G" any band that includes all frequencies allocated to both 2G and 3G services. We define as "Radial Disc-Cone Antenna RDCA" any discone antenna with a structure of radial wires. The RDCA was theoretically analyzed and software simulated with the purpose of computationally design a broadband model of it. As an application, a broadband RDCA for operation from 800 to 3,000 MHz, which include all 2G and 3G frequencies, was designed and an experimental model was constructed and tested. In order to evaluate the agreement between theory and practice, mathematically expressed measurement error bounds were computed.

## Introduction

In 1945, Kandoian invented the well-known discone antenna, that is a dipole made of a disc above a cone [1]. In 1953, Nail gave experimentally two naive relations for the discone dimensions [2].

In 1987, Rappaport designed discones using an N-type connector feed [3]. In 1993, Cooke studied a discone with a structure of radial wires [4]. In 2005, Kim et al. presented a double radial discone antenna for Ultra Wide-Band applications [5].

In this short paper we present an All-Band 2G+3G RDCA fed by an N-type/Female/50-Ohm connector.

## Research

The RDCA was theoretically analyzed as a group of identical filamentary V-dipoles with unequal arms connected in parallel. The dipoles recline on equiangular vertical  $\phi$ -planes around z-axis to form a disconical array. Fig.1A shows two such coplanar dipoles conformed with the apex angle  $\alpha$ . Each V-dipole has a total length  $L$  equal to the sum of arm lengths  $r$  and  $s$  plus the gap  $g$  between its terminals.

The simulation was based on a suite of developed visual tools which are supported by a fully analyzed, corrected and redeveloped edi-

tion of the original thin-wire computer program by J.H. Richmond [6].

Two arithmetic criteria were adopted for the broadband characterization of a model:

- (1) 50-Ohm VSWR lower than 2
- (2) Normalized radiation intensity  $U/U_{max}$  lower than 3 dB on the horizontal plane.

A visual application program was specifically developed to design a broadband radial discone with bare wires of diameter  $d$  embedded in free space when the wire conductivity, the type of feeding connector and the frequency band are given.

The program uses the model of a radial discone fed by an N-type connector shown in Fig.2. Starting with an appropriate combination of the relations given by [2]-[4] the program computes by iteration in terms of wavelength  $\lambda$ , the geometric characteristics  $r$ ,  $s$ ,  $g$ ,  $a$ , of the broadband model, just when the criteria are satisfied.

Fig.1B shows a Ground Plane Antenna GPA that was designed for reference and consists of equal number of cone radials  $s$  and a vertical monopole with height  $r$ .

As a practical application of the broadband design, the 2G+3G band from 800 to 2,500

MHz was selected to begin with and an experimental radial discone of copper wire fed by N-type connector was built, as shown in Fig.3.

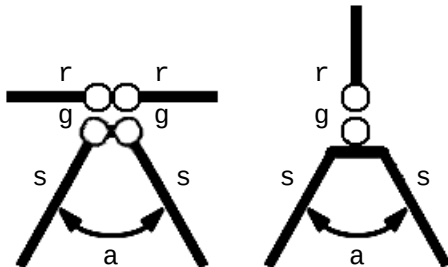


Fig.1: A - RDCA, B - GPA

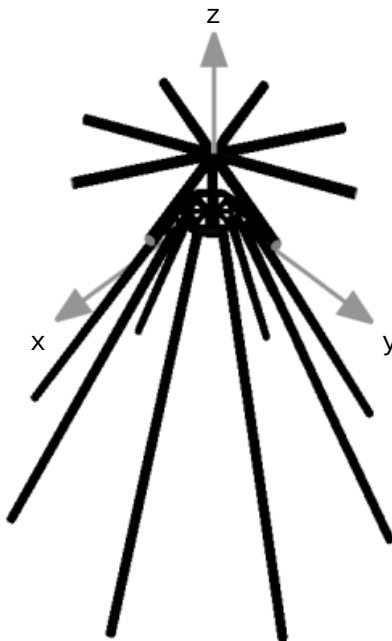


Fig.2: RDCA - Designed Model



In order to demonstrate the particular behavior of the experimental model, the 2G+3G band was divided as follows:

2G+3G Sub-Bands  
800 MHz – 2,500 MHz

Sub-Band	MHz
I	806 – 960
II	1,429 – 1,513
III	1,710 – 1,900
IV	1,910 – 2,025
V	2,110 – 2,170
VI	2,400 – 2,499

Our measurement system consists of an EM anechoic chamber, a network analyzer, a number of support instruments, a set of standard loads of factory accuracy and a constructed antenna rotation mechanism with a built hardware control unit of its step motor. The combined characteristics of system parts specify a measurement band from 600 to 1300 MHz, which overlaps with the 2G/3G band. Developed control software synchronizes the system and collects data using the IEEE-488 protocol.

A developed general mathematical method expresses the measurement error bounds. Another set of developed software applications processes the collected data and computes the error bounds.

**Results**

The consideration of radial discone as an array of at least eight 8 V-dipoles produces a theta-polarized vector radiation pattern with magnitude a surface almost by revolution around z-axis. So the radial discone has indeed on the horizontal plane xOy the basic properties of a vertically polarized almost omni-directional antenna, that is a fact that encouraged the design of a broadband model by using simulation.

The application of the broadband criteria to 2G/3G band resulted to the design of a RDCA with the following geometrical characteristics:

All-Band 2G+3G RDCA  
800 MHz – 3,000 MHz

Geometry		Units
d	1.5	[mm]
r	44	[mm]
g	6	[mm]
s	125	[mm]
a	60	[°]

The RDCA operates from 800 to 3,000 MHz, which exceeds that of 2G+3G band. The accordingly constructed experimental radial discone of Fig.3 should be implied with a constructional tolerance of ±0.5 mm and ±0.5°.

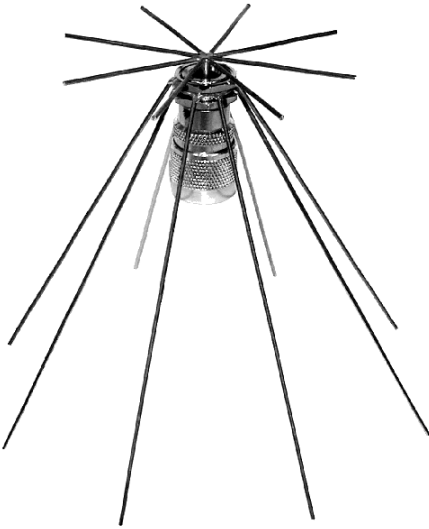


Fig.3: RDCA experimental model

The broadband model has a directivity from about  $-0.5$  to  $2.9$  dBd with slope angle between  $-65^\circ$  and  $+58^\circ$ , but the directivity gain on horizontal plane stays very close to the desirable value of  $0$  dBi, since it changes from  $-1$  to  $+1.7$  dBi only. Fig.4 shows that the predicted horizontal normalized radiation intensity remains below  $3$  dB indeed, while it stays above  $0$  dB relative to the reference antenna in all 2G+3G subbands indicated by the vertical gray strips, when both are fed by the same  $50\text{-}\Omega$  source.

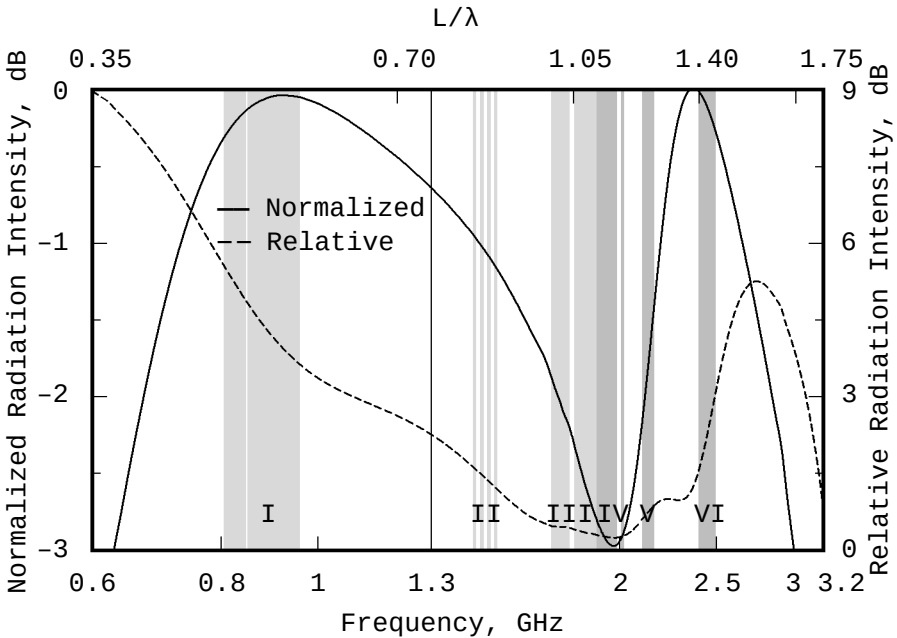


Fig.4: Predicted radiation intensity on horizontal plane.

Fig.5A shows the predicted normalized radiation patterns in dB at the center of each sub-band, which confirms the horizontal omni-directional radiation properties of the broadband model.

At the center frequency of 950 MHz of the measurement band, the predicted and measured radiation intensity on the three main cuts of the radiation pattern are in good

agreement, as shown in Fig. 5B.

This is made clearer by the measurement error bounds on a vertical plane as shown in Fig.6.

Fig.7 shows that the 50-Ohm VSWR predicted results for the broadband disccone are below 2 indeed and almost covered by the error bounds in the measurement band.

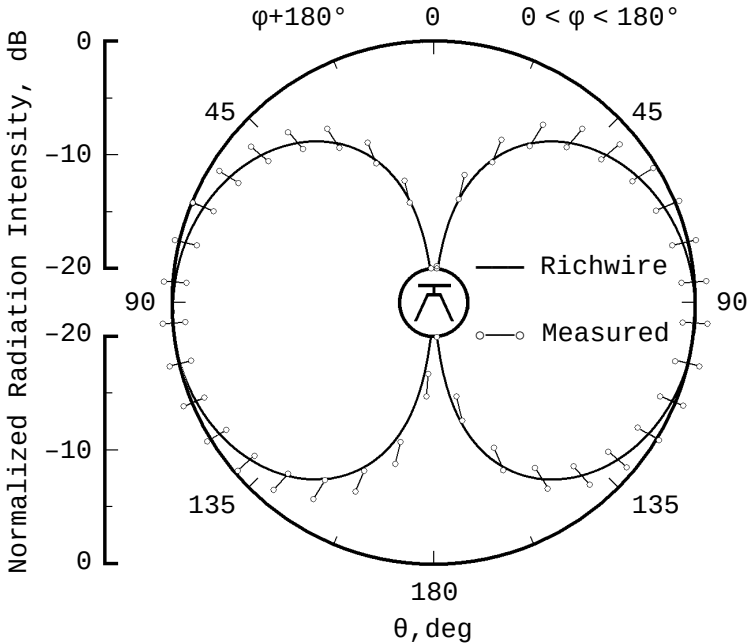


Fig.6: Measurement error bounds on a vertical plane

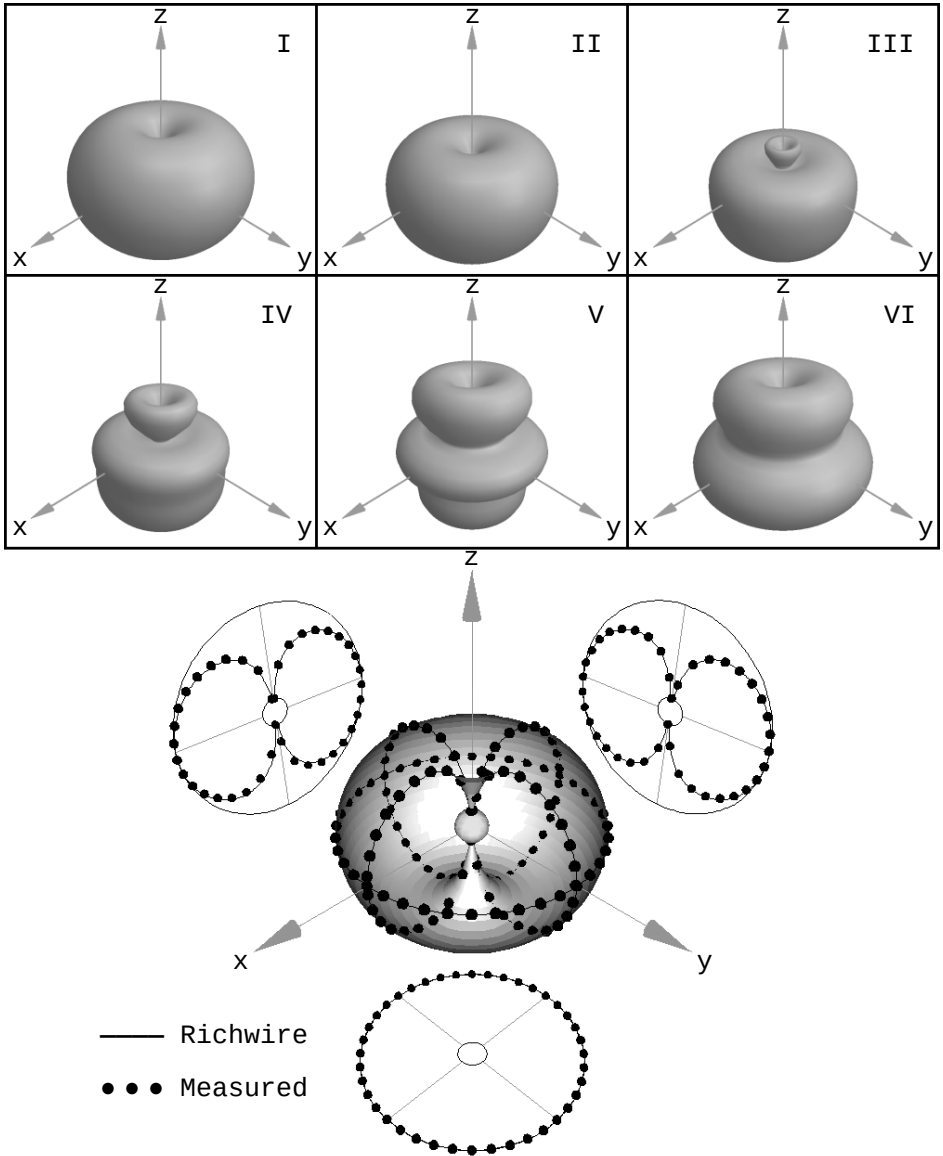


Fig.5: A (Up) Predicted normalized radiation intensity patterns at the center of each 2G+3G sub-band – B (Down) Normalized radiation intensity pattern at the center of measurements band

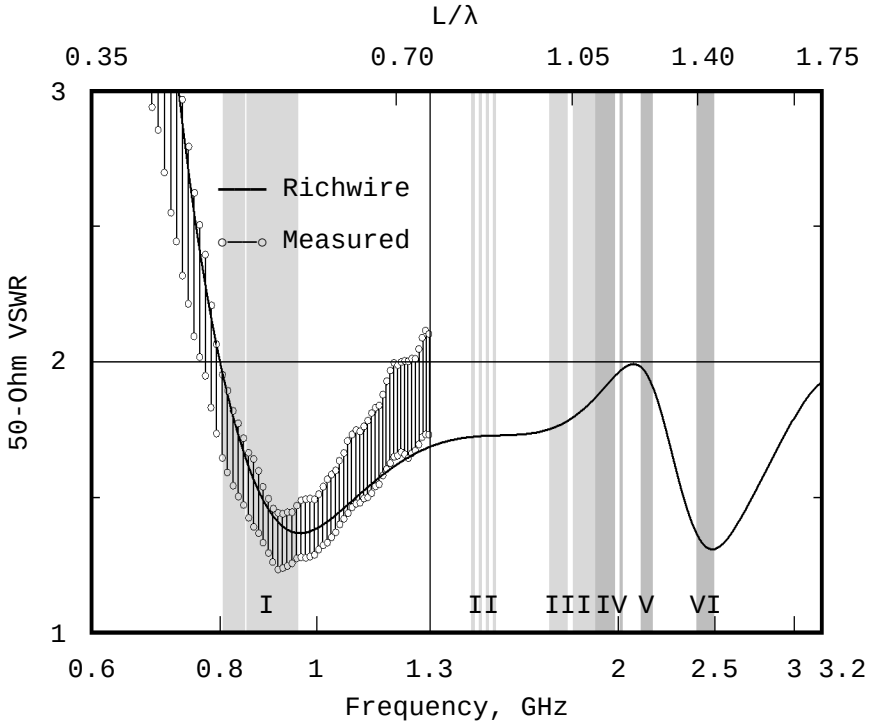


Fig.7: Standing wave ratio against frequency or ratio of total length to wavelength

**Conclusion**

Prediction and experimentation in the measurement band 600 MHz to 1,300 MHz proposes a successfully de-

signed, constructed, and measured Radial Disk Cone Antenna RDCA capable to serve All-Band 2G+3G applications from 800 MHz to 3,000 MHz.

**Credits**

The authors acknowledge Adamantios Diamantidis, SV7FSF, a former member of Computer Center – Network Administration Center at Democritus University of Thrace, now Systems Engineer, Systems Administrator (honorary), Berlin, Germany, who motivated the study of Discone Antennas on behalf of FSF Free Software Foundation followers on the purpose to be openly supported in their Wi-Fi activities.

## References

- [1] Kandoian A.G., "Three New Antenna Types and Their Applications", Proceedings of IRE, Vol. 34, February 1946, pp. 70W-75W
- [2] Nail J.J., "Designing Discone Antennas", Electronics, August 1953, pp. 167-169
- [3] Rappaport T.S., "Discone Design Using N-Connector Feed", IEEE Antennas and Propagation Society Newsletter, Vol. 30, February 1988, pp. 12-14
- [4] Cooke D.W., "The Skeleton Discone", The ARRL Antenna Compendium, Vol. 3, 1993, pp. 140-143
- [5] Kim K.H., Kim J.U. and Park S.O.: "An Ultrawide-Band Double Discone Antenna With the Tapered Cylindrical Wires", IEEE Transactions on Antennas and Propagation, Vol. 53, No. 10, October 2005, pp. 3403-3406
- [6] Richmond J.H., "Radiation and scattering by thin-wire structures in a homogeneous conducting medium", IEEE Transactions on Antennas and Propagation, Vol. 22, Issue 2, March 1974, p.365

## Preprint Versions

Broadband radial discone antenna:  
Design, application and measurements  
N. I. Yannopoulou, P. E. Zimourtopoulos, E. T. Sarris  
"<http://arxiv.org/abs/physics/0612043>"

## Follow-Up Research Paper

Not until now

## Previous Publication in FUNKTECHNIKPLUS # JOURNAL

None

### \* About The Authors

*Nikolitsa Yannopoulou*, SV7DMC, was born in Chania, Crete, Greece in 1969. She graduated in 1992 from Electrical Engineering at Democritus University of Thrace, Xanthi, Greece and since then she is with the Antennas Research Group (1992). She received the MEng degree with full marks in Microwaves in 2003 and her PhD degree in Antennas, in 2008 at Democritus University. Her research interests are in antenna theory, software, simulation, built, measurements and virtual laboratories.

yin@arg-at.eu – www.arg-at.eu

*Petros Zimourtopoulos*, SV7BAX, was born in Thessaloniki, Greece in 1950. 1976: Physics degree, 1978: Electronic Physics (*Radio-Electrology*) MSc and four courses to Mathematics degree, from Aristotle University of Thessaloniki. 1985: PhD in Antennas, from Electrical Engineering Department at Democritus University of Thrace. 1986: Lecturer; 1992: Assistant Professor; and since 1994, in permanent position, at Electrical Engineering and Computer Engineering Department of Democritus University. In 1985, he founded, and since then he leads, the independent Antennas Research Group.

pez@arg-at.eu – www.arg-at.eu

*Emmanuel Sarris* was born in Athens, Greece, 1945. Diploma of Physics, University of Athens, 1967. PhD in Space Physics (with J.A.Van Allen), University of Iowa, 1973. Post Doctoral Fellow, Applied Physics Laboratory of the Johns Hopkins University, 1974-76. Research Scientist, Max-Planck Institut, 1976-77. Professor of Electrodynamics at the Department of Electrical Engineering of the University of Thrace and Director of the Space Research Laboratory, 1977-2012. Experience: Space Plasma Electrodynamics; Design, Construction and Testing of Space Instruments and Systems; Satellite Communications; Satellite Remote Sensing. Co-I or P-I in the International Space Missions: Ulysses, Geotail, Interball-Aurora, Interball-Tail, Cluster, Spektr-R. Over 300 refereed publications and 1700 citations. COSPAR Plenary Council. Corresponding Member of the Academy of Athens, 2003. ESA and/or NASA Awards for outstanding contributions to the Ulysses, Geotail and Cluster Missions.

---

*This paper is licensed under a Creative Commons Attribution 4.0 International License – <https://creativecommons.org/licenses/by/4.0/>*

[ This Page Intentionally Left Blank ]



# Measurement Uncertainty in Network Analyzers: Differential Error Analysis of Error Models Part 1: Full One-Port Calibration

N.I. Yannopoulou, P.E. Zimourtopoulos \*

Antennas Research Group, Austria – Hellas [1, 2]  
EECE Dept, Democritus University of Thrace, Hellas [2]

## Abstract

An analytical method was developed to estimate errors in quantities depended on full one-port vector network analyzer (VNA) measurements using differentials and a complex differential error region (DER) was defined. To evaluate the method, differences instead of differentials were placed over a DER which was then analyzed and compared with another commonly used estimated error. Two real differential error intervals (DEIs) were defined by the greatest lower and least upper bounds of DER projections. To demonstrate the method, a typical device under test (DUT) was built and tested against frequency. Practically, a DER and its DEIs are solely based on manufacturer's data for standard loads and their uncertainties, measured values and their inaccuracies.

## Introduction

In full one-port measurements with a VNA of real characteristic impedance  $Z_0$ , a DUT with impedance  $Z$  has a reflection coefficient  $\rho$  defined by

$$\rho = (Z - Z_0)/(Z + Z_0)$$

and related to its measured value  $m$  by the bilinear transformation

$$\rho = (m - D)/[M(m - D) + R]$$

in terms of errors  $D$ ,  $M$  and  $R$  [1]. This transformation can be uniquely determined from given distinct  $\rho_n$ ,  $n = 1, 2, 3$  and respectively known  $m_k$ ,  $k = n$  [2].

## Research

We considered  $\rho_n$ ,  $m_k$  as the elements of given ordered triples  $(A, B, C)$ ,  $(a, b, c)$ , solved the resulting system and appropriately expressed its solution by

$$F = \sum cC(B - A)$$

$$D = \sum abC(A - B)/F$$

$$M = \sum c(B - A)/F$$

$$R = [\prod (A - B)(a - b)]/F^2$$

where  $\sum$  and  $\prod$  produce two more terms from the one shown, by rotation of the ordered triple elements. These errors were then considered

as depended on the independent variables  $\rho_n, m_k$ . Therefore, their differentials were expressed in the same manner by

$$dD = [\prod (a - b) \sum (B - C)BC \, dA + \sum (b - c)^2(B - A)(C - A)BC \, da] / F^2$$

$$dM = [\sum (a - b)(c - a)(B - C)^2 dA - \prod (A - B) \sum (b - c)da] / F^2$$

$$dR = \{ \sum [F + 2(a - b)B(A - C)] [(B - C)^2 dA \prod (a - b) - (b - c)^2 da \prod (A - B)] \} / F^3$$

After that, the differential of  $\rho$  was expressed by

$$d\rho = [-RdD - (m - D)^2 dM - (m - D)dR + Rdm] / [M(m - D) + R]^2$$

and was considered depended, through  $dD, dM$  and  $dR$ , on  $L = 7$  independent variables and their independent differentials:  $\rho_n, n = 1, 2, 3$  and  $m_k, k = n$  or  $k = 0$  with  $m_0 = m$ .

The developed expressions were mechanically verified using a developed software program for symbolic computations.

Manufacturer's data for standard loads used in full-one port VNA measurements are substituted in  $\rho_n$ , and for their uncertainties in  $d\rho_n$ . Since  $Z_0$  is real, the domain of each  $\rho_n$  is the closed unit circle [3]. For  $|\rho_n| = 0$  or 1, care must be exercised to restrict its differential value onto its domain. The VNA mea-

surements have specified bounded ranges for their modulus and argument, so that the domain of each  $m_k$  is a bounded circular annular with its center at the origin  $O$  of the complex plane. Measurement data are substituted in  $m_k$  and manufacturer's data for measurement inaccuracy in  $dm_k$ . Uncertainty and inaccuracy data outline domains for  $d\rho_n$  and  $dm_k$ . If  $z = |r|e^{j\varphi}$ , stands for any of the independent variables and  $dz$  for its differential then the contribution of  $dz$  to  $d\rho$  is a summation term of the form  $Wdz$ , with  $W = |U|e^{jV}$ , so that

$$Wdz = |U|e^{j(V + \varphi)} d|r| + |U|e^{j(V + \varphi + \pi/2)} |r| d\varphi$$

where  $W$  is in fact a known value of the respective partial derivative and  $d|r|, d\varphi$  are the independent real differentials of the complex  $dz$  in polar form. Each expression  $Wdz$  outlines a contour for a partial DER around  $O$ . If  $z \neq 0$ , the partial DER is a parallelogram with perpendicular sides  $d|r|$  and  $|r|d\varphi$ , stretched or contracted by  $|U|$  and rotated by  $(V + \varphi)$  around  $O$ . If  $z = \rho_n = 0$ , the partial DER is a circle with radius  $|U|d|r|$ . Accordingly, a DER is the sum of either  $L$  parallelograms or  $(L - 1)$  parallelograms and 1 circle. DER is

then a convex set with contour either a polygonal line with 4L vertices at most, or a piecewise curve composed of  $4(L - 1)$  line segments and  $4(L - 1)$  circular arcs at most. The greatest lower and least upper differential error bounds are the end-points of DEIs for the real and imaginary parts of  $\rho$  and result from the projections of DER for  $\rho$  on the coordinate axes. These conclusions can be generalized for any other quantity directly or indirectly depended on all, some or just one of the above independent variables and their differentials. Thus, the quantity has an L-term DER, where  $7 \geq L \geq 1$ . For example, the impedance Z of a DUT has the 7-term DER:

$$dZ = 2Z_0 d\rho / (1 - \rho)^2$$

## Results

All of the following data are specified by manufacturers of the parts for our measurement system. This system operates from 1 to 1300 MHz with 100 Hz PLL stability and consists of a type-N  $Z_0 = 50 \Omega$  network analyzer, a number of support instruments and a set of standard loads. The standards are: a short circuit A, a matching load B with reflection coefficient 0.029 and an open circuit C with reflection coefficient 0.99 and phase accuracy  $\pm 2^\circ$ . In the absence of manufacturer's

data for A we considered its uncertainty equal to that of C. So, the following values were substituted in the developed expressions:

$$A = -1, \quad 0 \leq d|A| \leq 0.01, \quad -180^\circ \leq d\varphi_A \leq -178^\circ \text{ or } 178^\circ \leq d\varphi_A \leq 180^\circ,$$

$$B = 0, \quad |dB| = 0.029,$$

$$C = 1, \quad -0.01 \leq d|C| \leq 0, \quad -2^\circ \leq d\varphi_C \leq +2^\circ$$

The annular domain for  $m_k$  of VNA is specified from 0 to -70 db in modulus and  $\pm 180$  degrees in argument. Measurements  $m_k$  result with a decimal floating point precision of 4 digits, for both modulus and argument. We consider the modulus and argument of  $dm_k$  equal to  $\pm 1/2$  of the unit in the last place of the corresponding mantissa in modulus and argument of  $m_k$ . Consequently, our system produces a DER, either for  $\rho$  or Z, as a sum of  $(L - 1) = 6$  parallelograms and 1 circle, with a contour of  $(4L + 4L) = 48$  vertices at most.

A suite of developed software applications: (i) controls the system and collects the data in terms of frequency using the IEEE-488 protocol, (ii) processes the collected data and computes the vertices of DER and the end-points of its DEIs (iii) sketches pictures of DER for  $\rho$  and Z in terms of the frequency steps and make a film using them as frames.

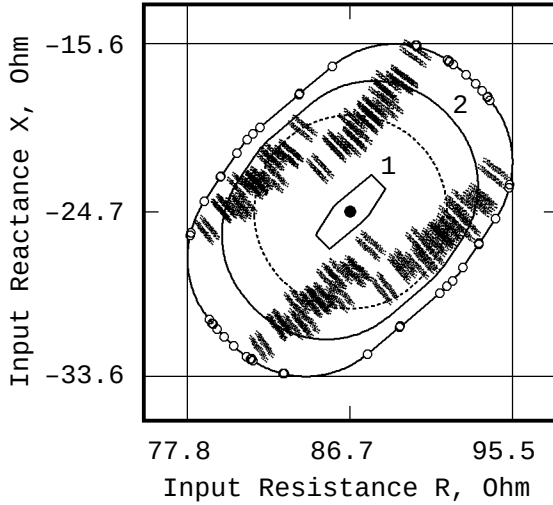


Fig.1: A typical DER for the impedance  $Z$

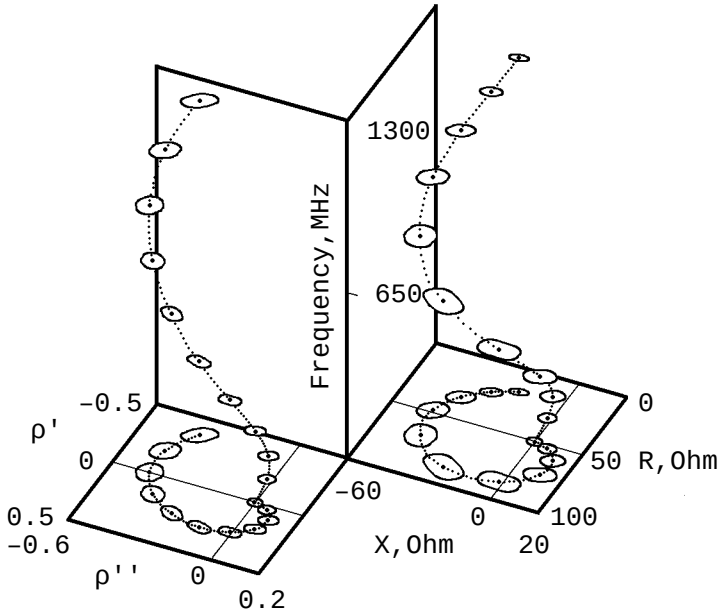


Fig.2: DER for the reflection coefficient  $\rho$  and for its associated impedance  $Z$  against frequency

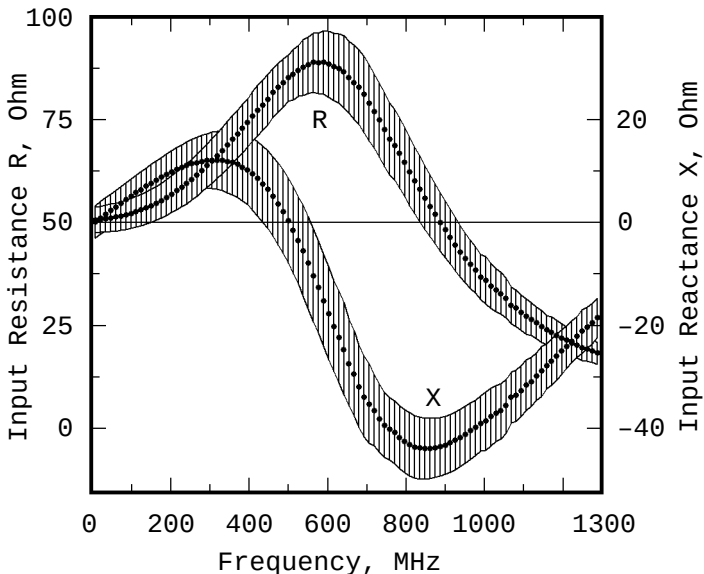


Fig.3: Greatest lower and least upper differential error bounds for resistance R and reactance X against frequency

A typical resistor with a nominal DC impedance of  $50 \Omega$   $\pm 20\%$  tolerance was soldered on a type-N base connector and enclosed in an aluminium box to serve as a simple DUT for testing its Z from 2 to 1289 MHz in 13 MHz steps. The center frequency  $f_c = 639$  MHz was chosen to reveal the details of the proposed method in Fig.1, where the contour of a typical DER for Z is outlined with small circles as its vertices. This contour surrounds that of the 4-terms DER due to inaccuracy of measurements (1) and that of 3-

terms DER for the uncertainty of loads (2). A properly circumscribed rectangle of DER shows graphically how the DEIs for R and X result. The commonly used error from the matching load only is shown as a dotted circle. This is in fact a 1-term DER which is surrounded from the contour of the DER by a factor of about 125% to 185% in all directions. Finally, in the same figure,  $2^{7 \times 2}$  differences  $\Delta Z$  resulting from the same  $d\rho_n$  and  $dm_k$ , dense enough to appear as stripes, were placed over DER to compare

them with differential  $dZ$  values. Notably, almost all of  $\Delta Z$  values are belong to DER while the computation time for these  $\Delta Z$  exceeds that for DER by more than one order of magnitude. To demonstrate the method, a set of selected DER frames for  $\rho$  and  $Z$  are shown in Fig.2, as beads on space curved filaments against frequency.

Finally, the computed DEIs for  $R$  and  $X$  are shown in Fig.3 against frequency.

### Conclusion

The proposed method may be efficiently used in the same way, to successfully estimate errors in any quantity depended on full one-port vector network analyzer measurements.

### References

- [1] Fitzpatrick J., "Error Models for Systems Measurement", Microwave Journal, 21, May 1978, pp. 63-66
- [2] Spiegel M.R., "Complex Variables with an introduction to Conformal Mapping and its applications", McGraw-Hill, 1974, p.203
- [3] Chipman R.A., "Transmission Lines", McGraw-Hill, 1968, p.137

### Preprint Versions

Differential Error Region of a Quantity Dependent on Full One-Port Network Analyser Measurements  
N. I. Yannopoulou, P. E. Zimourtopoulos  
"http://arxiv.org/abs/physics/0612049"

### Follow-Up Research Paper

Total Differential Errors in One-Port Network Analyzer Measurements with Application to Antenna Impedance  
N. Yannopoulou, P. Zimourtopoulos  
Radioengineering, June 2007, Volume 16, Number 2  
www.radioeng.cz/papers/2007-2.htm  
www.radioeng.cz/fulltexts/2008/08\_01\_01\_08.pdf

### Previous Publication in FUNKTECHNIKPLUS # JOURNAL

"All-Band 2G+3G Radial Disc-Cone Antennas: Design, Construction and Measurements", Issue 1, pp. 7-15

---

*This paper is licensed under a Creative Commons Attribution 4.0 International License – <https://creativecommons.org/licenses/by/4.0/>*

# Measurement Uncertainty in Network Analyzers: Differential Error Analysis of Error Models Part 2: Full Two-Port Calibration

N.I. Yannopoulou, P.E. Zimourtopoulos \*

Antennas Research Group, Austria – Hellas [1, 2]  
EECE Dept, Democritus University of Thrace, Hellas [2]

## Abstract

Since S-parameter measurements without uncertainty cannot claim any credibility, the uncertainties in full two-port Vector Network Analyzer (VNA) measurements were estimated using total complex differentials (Total Differential Errors). To express precisely a comparison relation between complex differential errors, their differential error regions (DERs) were used. To demonstrate the method in the most accurate case of a direct zero-length thru, practical results are presented for commonly used Z-parameters of a simple, two-port, DC resistive T-network, which was built and tested against frequency with a VNA measurement system extended by two lengthy transmission lines.

## Introduction

It is well known that in full two-port VNA measurements the S-parameters for a two-port Device Under Test (DUT) are given in terms of their 4 measurements  $m_{ij}$ ,  $i=1, 2$ ,  $j=1, 2$  by

$$S_{11} = \{[(m_{11} - D)/R][1 + (m_{22} - D')M'/R'] - L(m_{21} - X)(m_{12} - X')\}/(TT')\}/H \quad (1)$$

$$S_{21} = \{[1 + (m_{22} - D')(M' - L)/R'] (m_{21} - X)/T\}/H \quad (2)$$

$$H = [1 + (m_{11} - D)M/R][1 + (m_{22} - D')M'/R'] - LL'(m_{21} - X)(m_{12} - X')\}/(TT') \quad (3)$$

$S_{22}$ ,  $S_{12}$  have expressions that result from (1)-(2) by substituting  $i, j$  with  $j, i$  and  $D, M, R, L, T, X$  with  $D', M', R', T', L', X'$  and vice-versa [1]. These 12 quantities have been defined as system errors [2]. Stumper gave non-generalized expressions for the partial deviations of S-parameters due to calibration standard uncertainties, in 2003 [3]. Furthermore, the developed total differential errors for full one-port VNA measurements [4] are also not generalized in the two-port case. To the

best of the authors' knowledge, there are no analytical expressions for total differential errors in full two-port VNA measurements.

### Research

Since S-parameters are functions of 16 complex variables, their total differential errors were initially expressed as

$$\begin{aligned}
 dS_{11} = & \{ T T' (1 - MS_{11}) [R' + M' (m_{22} - D')] (dm_{11} - dD) \\
 & - RR' L (1 - L' S_{11}) [(m_{21} - X) (dm_{12} - dX') + (m_{12} - X') (dm_{21} - dX)] \\
 & + M' T T' [(m_{11} - D) (1 - MS_{11}) - RS_{11}] (dm_{22} - dD') \\
 & - T T' S_{11} (m_{11} - D) [R' + M' (m_{22} - D')] dM \\
 & + T T' (m_{22} - D') [(m_{11} - D) (1 - MS_{11}) - RS_{11}] dM' \\
 & - (R' L (1 - L' S_{11}) (m_{12} - X') (m_{21} - X) + \\
 & + T T' S_{11} [R' + M' (m_{22} - D')]) dR \\
 & - (RL (1 - L' S_{11}) (m_{12} - X') (m_{21} - X) \\
 & - T T' [(m_{11} - D) (1 - MS_{11}) - RS_{11}]) dR' \\
 & - RR' (m_{12} - X') (m_{21} - X) [(1 - L' S_{11}) dL - LS_{11} dL'] \\
 & + [(m_{11} - D) (1 - MS_{11}) - RS_{11}] [R' + M' (m_{22} - D')] \\
 & (T' dT + T dT') \} / P
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 dS_{21} = & \{ - MT T' S_{21} [R' + M' (m_{22} - D')] (dm_{11} - dD) \\
 & + RR' LL' S_{21} (m_{21} - X) (dm_{12} - dX') \\
 & + R \{ T' [R' + (m_{22} - D') (M' - L)] + R' LL' S_{21} (m_{12} - X') \} (dm_{21} - dX) \\
 & + T' (R (m_{21} - X) (M' - L) - M' TS_{21} [R + M (m_{11} - D)]) (dm_{22} - dD') \\
 & - T T' S_{21} (m_{11} - D) [R' + M' (m_{22} - D')] dM \\
 & + T' (m_{22} - D') (R (m_{21} - X) - TS_{21} [R + M (m_{11} - D)]) dM' \\
 & + \{ (m_{21} - X) (T' (m_{22} - D') (M' - L) + R' [T' + LL' S_{21} (m_{12} - X')]) \\
 & - T T' S_{21} [R' + M' (m_{22} - D')] \} dR \\
 & + (R (m_{21} - X) [T' + LL' S_{21} (m_{12} - X')]) \\
 & - T T' S_{21} [R + M (m_{11} - D)]) dR' \\
 & + R (m_{21} - X) [R' L' S_{21} (m_{12} - X') - T' (m_{22} - D')] dL \\
 & + RR' LS_{21} (m_{12} - X') (m_{21} - X) dL' \\
 & - T' S_{21} [R + M (m_{11} - D)] [R' + M' (m_{22} - D')] dT \\
 & + (R (m_{21} - X) [R' + (m_{22} - D') (M' - L)]) \\
 & - TS_{21} [R + M (m_{11} - D)] [R' + M' (m_{22} - D')] dT' \} / P
 \end{aligned} \tag{5}$$



$$P = T T' [R' + M' (m_{22} - D')] [R + M (m_{11} - D)] - R R' L L' (m_{12} - X') (m_{21} - X) \quad (6)$$

$dS_{22}$  and  $dS_{12}$  resulted from (4), (5) with the mentioned substitutions.  $X, X'$  errors stand for crosstalk measurements.  $D, M, R (D', M', R')$  errors are uniquely determined in terms of 3 standard loads  $A, B, C (A', B', C')$  and their 3 measurements  $a, b, c (a', b', c')$ , by full one-port VNA measurements, so the number of independent complex variables increases from 16 to 22.  $L, T (L', T')$  errors are accurately determined after the replacement of DUT with a direct thru (or approximately, if an adapter is used instead) in terms of new measurements  $t_{11}, t_{21} (t_{22}, t_{12})$  and of previously found quantities. Their expressions were appropriately stated as

$$L = [\sum (ab + ct_{11})C(B - A)]/E \quad (7)$$

$$T = (t_{21} - X) [\prod (A - B)(a - b)] / (E [\sum cC(B - A)]) \quad (8)$$

$$E = \sum (ab + ct_{11})(B - A) \quad (9)$$

where  $\sum$  and  $\prod$  produce two more terms, from the given one, by cyclic rotation of the letters  $a, b, c (a', b', c')$  or  $A, B, C (A', B', C')$ . In this way, each S-parameter has as total differential error  $dS$ , a sum of 22 differential terms:

16 due to measurement inaccuracies  $dm_{ij}, dX, dX', dt_{ij}, da, db, dc, da', db', dc'$  and 6 due to standard uncertainties given by their manufacturer  $dA, dB, dC, dA', dB', dC'$ . The expressions for  $dD, dM, dR (dD', dM', dR')$  are known [4]. The expressions for the rest of differential errors were developed as

$$dL = \{ \sum (B - C)(b - t_{11})(c - t_{11}) [(B - C)(b - a)(c - a)dA - (b - c)(B - A)(C - A)da] + [\prod (A - B)(a - b)] dt_{11} \} / E^2 \quad (10)$$

$$dT = \{ \sum (t_{21} - X)(b - c)(B - C) [(t_{11} - c)(b - a)^2 B(A^2 + C^2) + (b - t_{11})(c - a)^2 C(A^2 + B^2) - 2ABC(b - c)(t_{11}(b + c - 2a) - bc + a^2)] [(B - C)(b - a)(c - a)dA - (b - c)(B - A)(C - A)da] \} / (E^2 [\sum cC(B - A)]^2) + [\prod (A - B)(a - b)] \{ [(t_{21} - X) \sum a(B - C) / E] dt_{11} + dt_{21} - dX \} / (E [\sum cC(B - A)]) \quad (11)$$

Each complex differential error defines a Differential Error Region (DER) on the complex plane with projections to coordinate axes the Differential Error Intervals (DEIs) [4]. Obviously, any

quantity differentially dependent on the above variables has also a DER. For example, after another correction to the given S to Z-parameters relations [5], the Z-DERs are resulted from

$$dZ_{11} = 2Z_0[(1 - S_{22})^2 dS_{11} + (1 - S_{22})S_{21} dS_{12} + (1 - S_{22})S_{12} dS_{21} + S_{12}S_{21} dS_{22}] / [(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}]^2 \quad (12)$$

$$dZ_{21} = 2Z_0[(1 - S_{22})S_{21} dS_{11} + S_{21}^2 dS_{12} + (1 - S_{11})(1 - S_{22}) dS_{21} + (1 - S_{11})S_{21} dS_{22}] / [(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}]^2 \quad (13)$$

while  $dZ_{22}$ ,  $dZ_{12}$  result from (12), (13) by application of the mentioned substitutions.

S-DER is a sum of 20 parallelograms and 2 circles, with a contour of 160 vertices at most [4].

### Results

Six calibration standards, in pairs of opposite sex, were used and their manufacturers' data were substituted in the developed expressions:

$A = -1 = A'$ ,  $0 \leq d|A| = d|A'| \leq 0.01$ ,  $-180^\circ \leq d\varphi_A = d\varphi_{A'} \leq -178^\circ$  or  $178^\circ \leq d\varphi_A = d\varphi_{A'} \leq 180^\circ$ ,

$B = 0 = B'$ ,  $|dB| = 0.029 = |dB'|$ ,

$C = 1 = C'$ ,  $-0.01 \leq d|C| = d|C'| \leq 0$  and  $-2^\circ \leq d\varphi_C = d\varphi_{C'} \leq +2^\circ$ .

The inaccuracy of any VNA measurement was conservatively considered as a symmetric interval defined by just 1 unit in the last place of the corresponding mantissa, both in modulus and argument. Consequently, each

To demonstrate the method, a typical T-network of common resistors with nominal DC values  $Z_1 = 24.2 \Omega$ ,  $Z_2 = 120 \Omega$  for the horizontal arms and  $Z_{12} = 1.1 \Omega$  for the vertical arm, were soldered on type-N base connectors of opposite sex and enclosed in an aluminium box, to form a two-port DUT.

The VNA measurement system was extended by two transmission lines of 3.66 m and 14 m, respectively, up to the DUT. The DUT was tested from 2 to 1289 MHz in 13 MHz steps. The frequency 1003 MHz was selected to illustrate the proposed method for S-DERs shown in Fig. 1.

To study the total differential error,  $dS$  was expressed as  $dU + dI$ , where  $dU$  is due to the uncertainty of 6 standards and  $dI$  to the inaccuracy of 16 measurements. The contribution of these, conservatively considered measurement inaccuracies to the total differential error is as much significant as the uncertainties of standard loads are. For example, computations for  $S_{12}$  over the whole measurement band show that  $\max|dU|$  and  $\max|dI|$  contribute  $\sim 35\text{-}80\%$  and  $\sim 25\text{-}70\%$  to  $\max|dS_{12}|$ , respectively. In addition, Fig. 1 shows how the projections of each S-DER result its real and imaginary DEI. To display the variation of S-DER against frequency, a number of selected S-DER frames are shown in Fig. 2 as beads on a space-curved filament. It is worth mentioning that  $S_{11}$ -DER ( $S_{22}$ -DER) was greater than it resulted from appropriately organized full one-port measurements, as it was expected. Finally, the computed Z-DEIs are shown in Fig. 3, along with their LF Z-values.

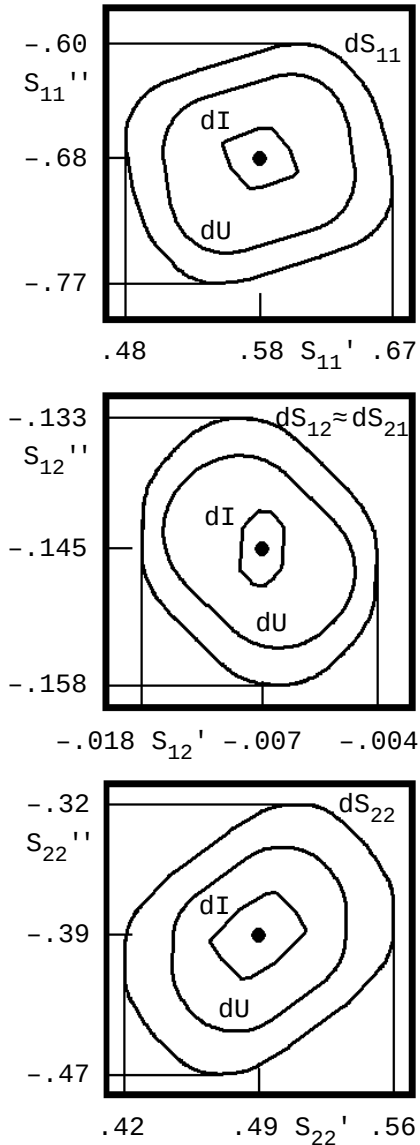


Fig.1: S-DERs at 1003 MHz

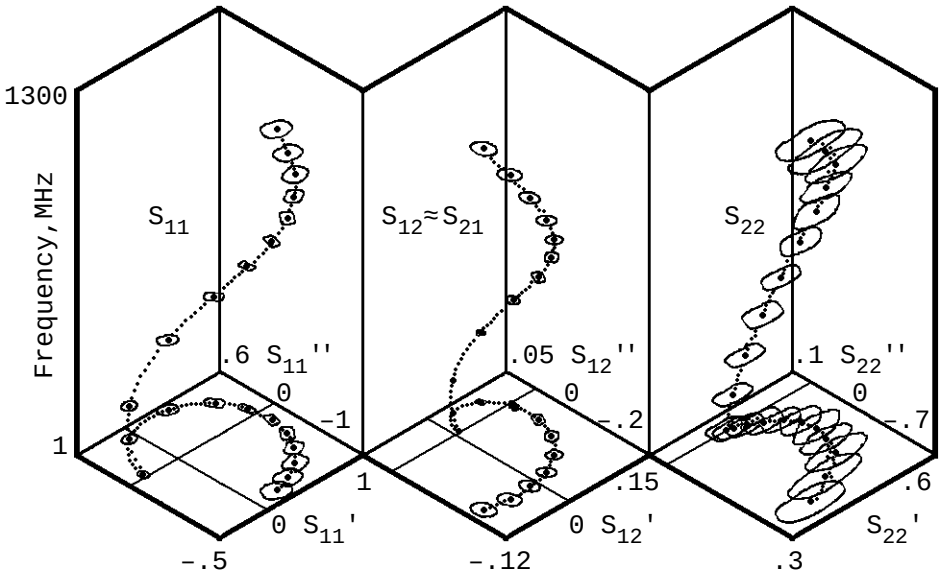


Fig.2: S-DERs against frequency

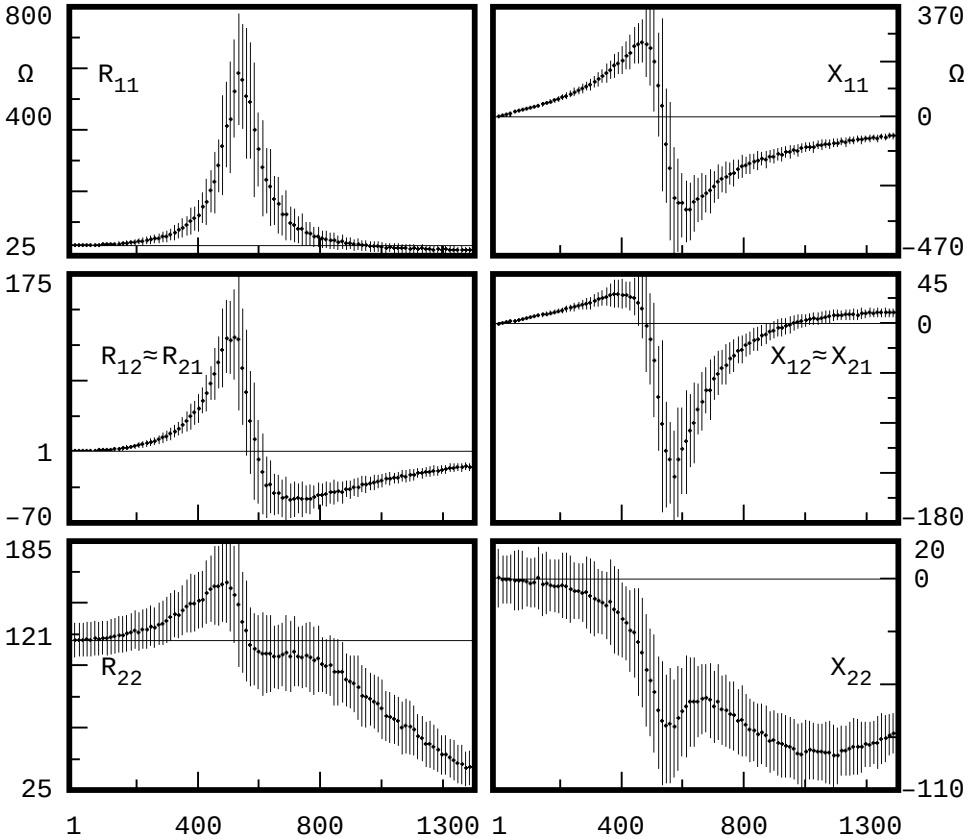


Fig.3: Z-DEIs against frequency

**Conclusion**

The proposed method may be efficiently used to estimate

uncertainties in any case where the process equations (1), (2) and (4), (5) can find application.

## References

- [1] Ballo D., "Network Analyzer Basics", Hewlett-Packard Company, 1998, p. 58
- [2] Fitzpatrick J., "Error Models for Systems Measurement", Microwave Journal, 21, May 1978, pp. 63-66
- [3] Stumper U., "Influence of TMS0 calibration standards uncertainties on VNA S-parameters measurements", IEEE Transactions on Instrumentation and Measurements, Vol. 52, No. 2, April 2003, pp. 311-315
- [4] Yannopoulou N., Zimourtopoulos P., "Total Differential Errors in One-Port Network Analyzer Measurements with Application to Antenna Impedance", arXiv:physics/0703204, Radioengineering, Vol. 16, No. 2, June 2007, pp. 1-8
- [5] Beatty R.W., Kerns D.M., "Relationships between Different Kinds of Network Parameters, Not Assuming Reciprocity or Equality of the Waveguide or Transmission Line Characteristic Impedances", Proceedings of the IEEE, Vol. 52, Issue 1, January 1964, p. 84

## Preprint Versions

Total Differential Errors in One-Port Network Analyzer Measurements with Application to Antenna Impedance  
Nikolitsa Yannopoulou, Petros Zimourtopoulos  
"http://arxiv.org/abs/physics/0703204"

## Follow-Up Research Paper

S-Parameter Uncertainties in Network Analyzer Measurements with Application to Antenna Patterns  
N. Yannopoulou, P. Zimourtopoulos  
Radioengineering, April 2008, Volume 17, Number 1  
www.radioeng.cz/papers/2008-1.htm  
www.radioeng.cz/fulltexts/2008/08\_01\_01\_08.pdf

## Previous Publication in FUNKTECHNIKPLUS # JOURNAL

"Measurement Uncertainty in Network Analyzers: Differential Error Analysis of Error Models Part 1: Full One-Port Calibration", Issue 1, pp. 17-22

---

*This paper is licensed under a Creative Commons Attribution 4.0 International License – <https://creativecommons.org/licenses/by/4.0/>*

[ This Page Intentionally Left Blank ]

In case of any doubt,  
download the genuine papers from  
**[genuine.ftpj.otoiser.org](http://genuine.ftpj.otoiser.org)**

## FRONT COVER VIGNETTE

A faded synthesis of an anthemion rooted in a meandros

The thirteen-leaf is a symbol for a life tree leaf.  
"Herakles and Kerberos", ca. 530–500 BC,  
by Paseas, the Kerberos Painter,  
Museum of Fine Arts, Boston.

[www.mfa.org/collections/object/plate-153852](http://www.mfa.org/collections/object/plate-153852)

The simple meandros is a symbol for eternal immortality.  
"Warrior with a phiale", ca. 480–460 BC,  
by Berliner Maler,  
Museo Archeologico Regionale "Antonio Salinas" di Palermo.

[commons.wikimedia.org/wiki/File:Warrior\\_MAR\\_Palermo\\_NI2134.jpg](https://commons.wikimedia.org/wiki/File:Warrior_MAR_Palermo_NI2134.jpg)