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 * The Constitution of Greece, Article: 12(3) 2008:
   www.hellenicparliament.gr/en/Vouli–ton–Ellinon/To–Politevma

 * The Hellenic Supreme Court of Civil and Penal Law:
   www.areiospagos.gr/en/ — Court Rulings:Civil|A1|511|2008

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The Very First Ever Made
3D/4D Virtual Laboratory for Antennas

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Abstract

Based on the experience we have gained so far, as independent reviewers, we thought that may be proved useful to publicly share with the interested author, especially the young one, some practical implementations of our ideas for the interactive representation of data using 3D/4D movement and animation, in an attempt to motivate and support him in the development of similar dynamic presentations, when he is looking for a way to locate the stronger aspects of his research results in order to prepare a clear, most appropriate for publication, static presentation figure. For this purpose, we selected to demonstrate a number of presentations, from the simplest to the most complicated, concerning well-known antenna issues with rather hard to imagine details, as it happens perhaps in cases involving Spherical Coordinates and Polarization, which we created to enrich the very first ever made Virtual Laboratories of Antennas, that we distribute over the Open Internet through our website Virtual Antennas. These presentations were developed in a general way, without using antenna simulators, to handle output text and image data from third-party CAS Computer Algebra Systems, such as the Mathematica commercial software we use or the Maxima FLOSS we track its evolution.

Introduction

The very first ever made Virtual Laboratories of Antennas, which are available on the Internet through Virtual Antennas website [1], are exclusively based on our alternative form of Antenna Theory [2], and are founded, by applying the learning by teaching method to Antennas education [3], in order to support the quick comprehension of 3D space matters unavoidably related to antennas. The initial material of Virtual Antennas was prepared during the years 1996-1997 and presented to Antenna students during the spring seme-

ster of 1998 [4]. On 1999, the existence of this visual material was announced to the first ever appeared on the web EMLIB Electromagnetics Library, maintained those years by Jet Propulsion Laboratory of NASA [5]. On the same year, all of the material was approved for inclusion to MathSource repository of Mathematica [6].

Since 2000 and until today, the Virtual Antennas material is constantly increased and improved. Moreover, a number of websites either use this material or suggest its use [7]. On 2009, a sample of its current development state was approved for publication by the Wolfram Demonstrations Project [8], an event that was announced in our Creative Commons Network web pages [9].

During the last years, our voluntary reviewing work revealed some needs of potential authors -especially the young ones- related to the presentation of their research results, which perhaps may be confronted by similar to our interactive presentation techniques, and thus we decided to present in this paper our continuing work for the Virtual Laboratories of Antennas.

Presentation Development and Use

The implementation of a presentation idea demands the expression of the theoretical idea formulation in the CAS language we select to produce output data in the appropriate file format for movement or/and animation (we would like to emphasize that in the case of Mathematica we use, while we abandoned the package written by Novak [10], we still run the one written by Donley [11]). These files are: (a) WRL (WoRLd: Open by Web3D Consortium), a plain text file with a known structure described in the VRML Virtual Reality Modeling Language [12] for a visual object, (b) a bunch of images in a selected non-destructive format; usually BMP, and (c) NBP text file (Note Book Player: Mathematica proprietary). After that, the development of the presentation requires: (a) either a text editor to correct bugs or to add special features into WRL output file, (b) a graphics editor to align images or correct blemishes, as well as, a video editor to handle these images as frames of the final movie file in a selected format, usually AVI (Audio Visual Interleave: Microsoft proprietary). It is worth to notice that accord-

ing to our experience, the non-computing time needed: (a) to design the idea implementation, has been reduced by the years, from a couple of weeks at the beginning to just a couple of days now, and (b) to develop the presentation, after a tedious, routine work of editing each frame, varies considerably according to the frame theme, e.g. from 1.5 minutes for that in Fig. 7, to 10 minutes of Fig. 10.

The software needed for presentations to work under MS Windows is: a web browser (MS-IE 3.02r+), a VRML viewer add-on to that browser (World-View 2.1), an AVI player (Mplayer2), and the Mathematica 7 Notebook Player.

3D/4D Presentation Samples

In order to abbreviate the figure descriptions in the following, we have to notice that: (a) for every animation presentation, a typical full-window screen capture of the recommended AVI player is shown after its pause button was pressed, (b) for every movement presentation, a typical full-window screen capture of the recommended online WRL viewer, with a cursor perhaps to indicate the existence of an additional feature, and (c) for both presentation types, the basic colors (R, G, B) are used in that order to correspond either to the CCS Cartesian Coordinate System (x, y, z) axes, (xoy, yoz, zox) planes, and its unit vectors, or to the SCS Spherical Coordinate System (r-radius, theta-semi-circle, phi-circle) curves, (r-sphere, theta-semi-cone, phi-semi-plane) surfaces, and its unit vectors, while the same color correspondence holds for the radiation pattern cuts by the mentioned CCS planes or SCS surfaces.

After that, short descriptions of the samples, are following, while all of these presentations will be always available in authors' group repositories in Virtual Antennas [13] and Google Code Repository [14].

Fig. 1 shows 1 frame out of 12 of an AVI repeated-forever animation for the 3D plane-time presentation of the considered as time-harmonic sinusoidal current wave amplitude, on a 3 wavelengths portion of a long thin wire loop antenna terminated on some complex impedance. The propagated current wave p is decomposed in the dual couples: (incident wave i, reflected wave r) and (standing wave s, transmitted wave t), while the imposed letters and arrows on the figure indicate these current waves as well as the direction of their motion.

Fig. 2 shows a screen-shot of a WRL presentation with 5 additional predefined view points of 12 frames animation, as the cursor indicates, of a 4D space-time asynchronous movement, from a view-point in the 1st-octant. In essence, this presentation is an alternative geometric meaning of the considered as standing wave current on an adequate length of a thin wire dipole antenna.

Fig. 3 shows a screen-shot of a WRL presentation of 4D space-time asynchronous movement from a view point in the 1st-octant, of the 3D difference volume element formed by adjacent SCS coordinate surfaces and lying between the 1st and 2nd octant.

Fig. 4 shows a screen-shot of a WRL presentation of a 4D space-time asynchronous movement from a view-point in the 1st-octant, for the triples of SCS unit vectors in the shown directions, that is 13 triples on the CCS main-planes, as well as, 2 undefined triples on the irregular z-axis of the CCS-to-SCS transformation.

Fig. 5 shows a screen-shot of a WRL presentation of a 4D space-time asynchronous movement from a view-point in the 1st-octant, for all 3 SCS surfaces, curves, and unit vectors in a direction of the 1st- octant.

Fig. 6 shows 1 frame out of 37 of an AVI animation for the 4D plane-time presentation of a SCS phi-circle curve as the intersection between 2 SCS surfaces, that is of a r-sphere and a theta-semi-cone, while theta angle -which changes from 0° to 180°- has the value of 50°. Different colors illustrate the inner and outer surfaces of the r-sphere and a cut off spherical section between adjacent meridians permit us to see the interior of that sphere.

Fig. 7 shows a screen-shot of a WRL presentation of a 4D space-time asynchronous movement for three different polarizations at three different points: linear, circular and elliptical on each of the coordinate axes x, y and z respectively to emphasize that, in general, the polarization of an antenna is not constant.

Fig. 8 shows 1 frame out of 20 of an AVI animation for the 4D space-time presentation of the non-linearly CCW polarized time-harmonic real electric field E, from an antenna, as well as, its decomposition to two linearly polarized time-harmonic real electromagnetic fields, as they are uniquely defined by the constant of time linearly independent space vectors E_c and E_s.
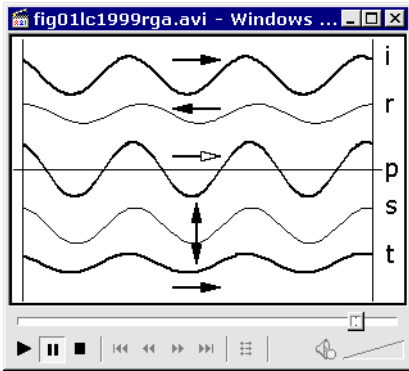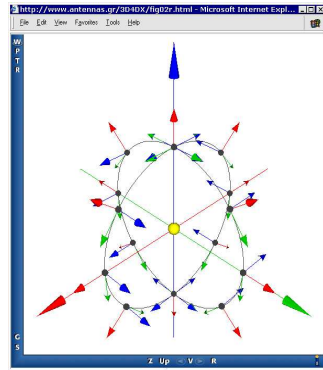
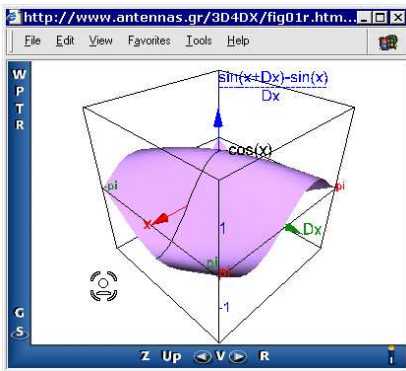Fig. 1: Current waves

Fig. 4: SCS: Unit vectors

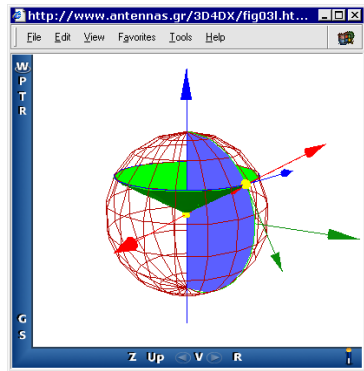Fig. 2: Standing current-
voltage waves relation

Fig. 5: Coordinate curves,
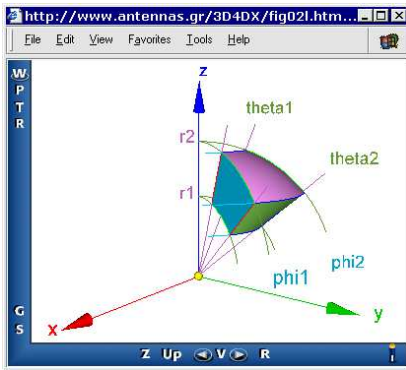surfaces and unit vectors

Fig. 3: SCS: Volume element
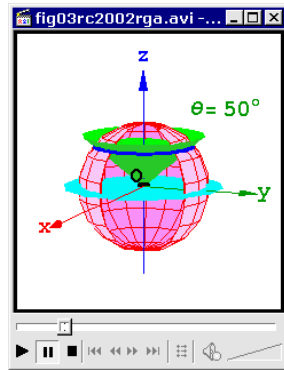
Fig. 6: Coordinate surfaces

Fig. 9 shows the last fra-me out of 23 of an AVI anima-tion for the 4D space-time presentation of the path traced by the electric far-field arrow tip, which is CW elliptically polarized in the direction of y-axis.

Fig. 10 shows 1 frame out of 19 of an AVI for the 4D space-time presentation of a normalized CCW elliptically polarized electromagnetic far -field in the shown direction of propagation, from a point of view in the 1st octant of an observer mirrored to (r, theta) plane.
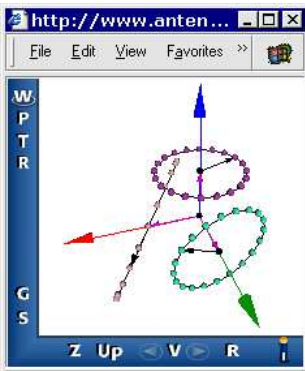
Fig. 7: Polarization:
Directional dependence

Fig. 9: Polarization:
CW elliptical

Fig. 8: Polarization:
Non-linear CCW

Fig. 10: Polarization:
Far-Field CCW elliptical.

Fig. 11 shows 3 frames out of 73 of an AVI for the 4D space-time presentation of the direction dependency of antenna polarization from a center fed short crossed dipoles on y and z axes, which have a current phase difference of 90°. The three different polarizations are shown with their orientation for an observer on the xOy plane (theta = 90° and in three different phi angles of 0°, 240°and 270°.
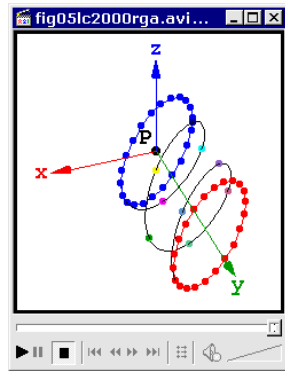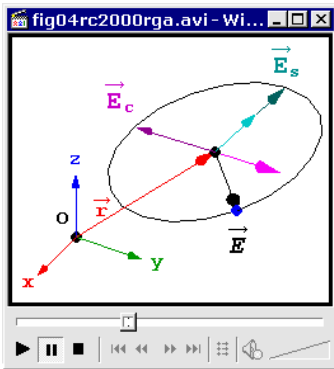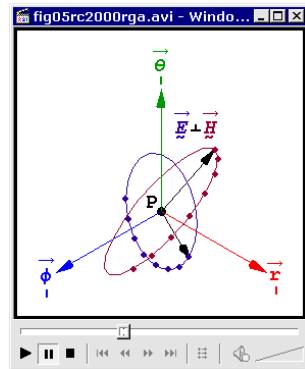
Fig. 12 shows 4 frames out 38 of an AVI animation for the 4D space-time presentation of the space radiation pattern and its three main-planes cuts, for the specific case of an antenna consisting of 2 dipoles, with the same direction of the unit vector (0.2, 0.4 0.894), each of 2.4λ length long, with their centers placed 0.25λ apart on an axis with a unit directional vector (0.3 0.5 0.812), fed with a 30° current phase difference.

Fig. 13 shows a screenshot of a WRL presentation with 1 additional predefined view point of 12 frames animation possibility, as the cursor indicates, of a 4D space-time asynchronous movement, from a view-point in the 1st-octant. This presentation concerns the interior of an anechoic chamber and imitates a discone antenna

rotation in order to visualize the way by which the measurement of a plane-cut of its radiation pattern is accomplished.
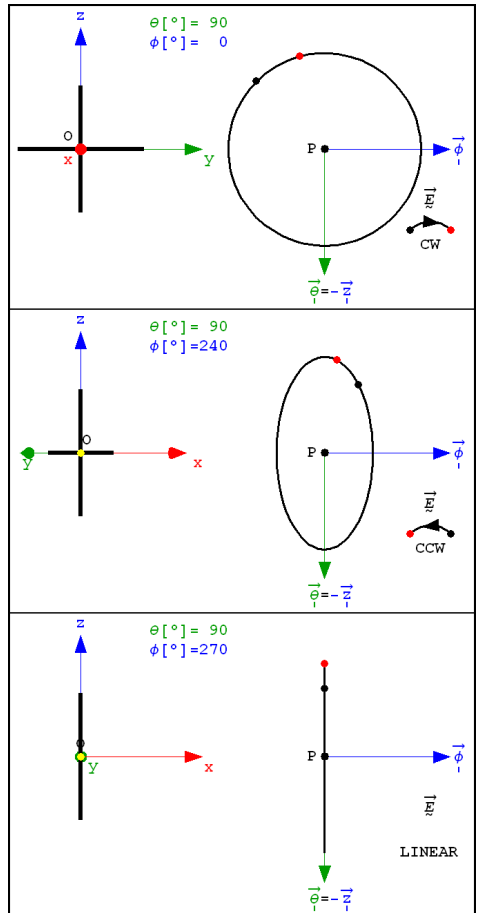
Fig. 11: Polarization: CW Circular – CCW Elliptical – Linear

Fig. 14 shows a sophisticated combination of a movement with an animation presentation, which in fact is an application that was developed using the version 7 of Mathematica, in a way that permits the definition of any (theta, phi) direction in space for a dipole of variable length. Notably, this application has all the available VRML movement features, with 6 predefined view points, while 3 different animations can be run simultaneously. In addition, the mesh, opacity, rendering goal, and the evaluation step for the radiation pattern can be also defined.

Fig. 15 shows the last frame out of 100 of an AVI for the 4D space-time presentation of a detailed study for some of the linear dipole characteristics regarding the plane and space radiation patterns, the direction from the dipole axis of their first common maximum, the directivity, and the input radiation resistance; this really is a 4-tuple of frames.
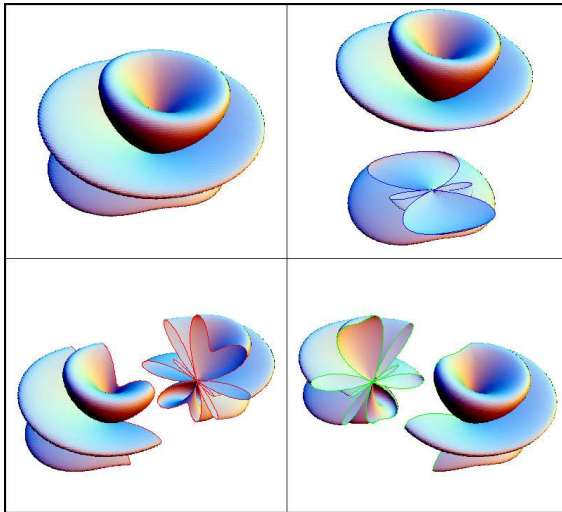
Fig. 12: Radiation Pattern: 3D and its 2D main-plane cuts
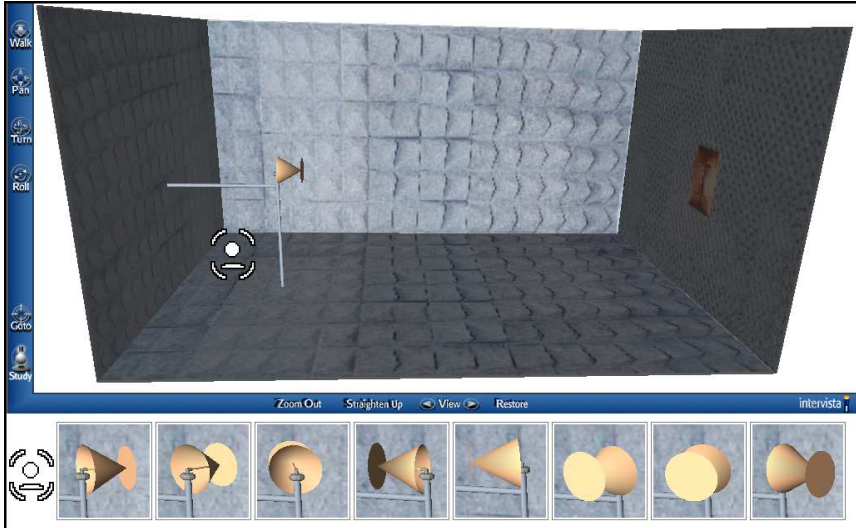
Fig. 13: Anechoic chamber – discone antenna
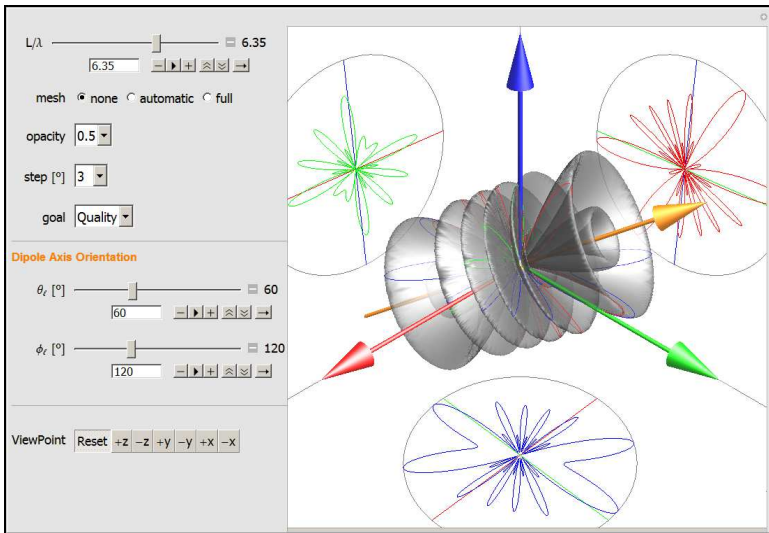
Fig. 14 Radiation Pattern: A composite 3D/4D presentation
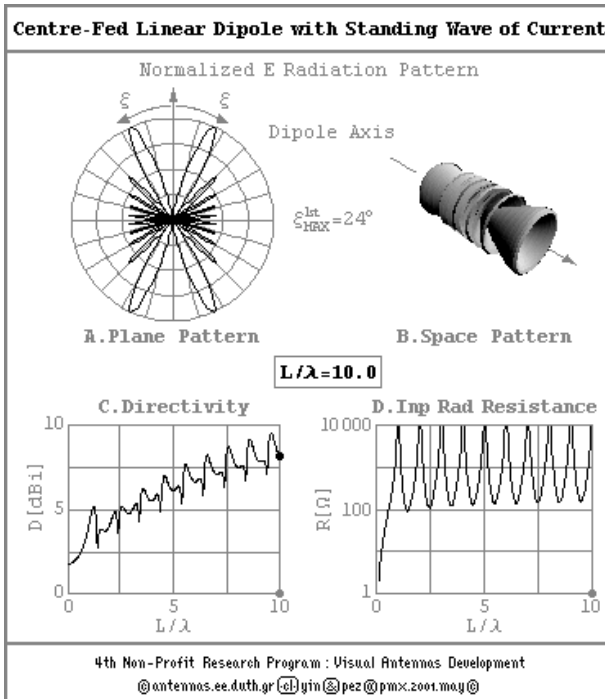
Fig. 15: Antenna characteristics

Conclusion

The Antennas Research Group (ARG) carries on for the enhancement of this low-cost Virtual Laboratory with new material placed to web site www.antennas.gr [13], as well as to Google Code Repository [14], and Fig. 13 and Fig. 14 are only two of many more possible examples.

The evolution on computer systems in recent years has increased the demand for new teaching, learning and re-searching methodologies, which this paper supports by describing available material for a web-based Virtual Laboratory for Antennas that provides interactive computer aided visualization in various Antenna Theory topics.

The main goal is constantly attained since this continuously developed material cannot be produced by the means of the conventional literature.

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"Support of Interactive 3D/4D Presentations by the
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Near-field Method in Solving Inverse Scattering Problem of Spherical Electromagnetic Waves in Chiral media

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Abstract

We have developed two methods to study an inverse scattering problem of electromagnetic waves in chiral media, for a spherical scatterer perfect conductor. The first called Far-field inverse problem was described analytical in our previous work. The second called Near-field method is given in detail in the present paper. It is a geometrical method based on the scattered fields. Using Near-field experiments, in which the scattered field is measured at the source, we solve the corresponding inverse scattering problem that is to determine the coordinates of the center and the radius of the spherical scatterer.

Introduction

We study an inverse scattering problem of electromagnetic waves with time harmonic dependence applying the method of Near-field [1]-[3]. Spherical electromagnetic waves generated by a point source incident on a spherical scatterer perfect conductor in a chiral media.

Knowing the incident and scattered wave fields in the inverse problem we are looking for the coordinates of the sphere center and its radius.

The corresponding problem is solved by geometrical methods using either energy scattering cross-section, or the scattered fields. In the first case we refer to the inverse Far-field problem [4] and in the second case to a Near-field inverse problem.

The chiral materials exhibit the phenomenon of optical activity i.e., the phenomenon that in which the plane of polarizations of linearly polarized light is rotated as the light passes through an optically active medium.

The chirality is a property which is often found in nature and reflects the asymmetry in the spatial inversion. An electromagnetic wave into a chiral media is analyzed in a counter-clockwise

(LCP) and a clockwise (RCP) Beltrami field. Using vector spherical harmonic functions and appropriate expansions of the Beltrami fields and by extension the development of spherical electromagnetic waves in spherical wave functions, we calculate the exact solution of the scattering problem for a spherical perfect conductor in chiral media, and the corresponding back scattering [3]-[4].

In the inverse problem we calculate the scattered Near-field, when the source is in position $\mathbf{r} = \mathbf{r_0}$. A similar problem for achiral materials has been studied in the works [1], [2].

Statement of the problem

Consider a point source at the position $\mathbf{r_0}$ that produces spherical electromagnetic waves in a chiral media near a scatterer Ω^-, perfect conductor, i.e. the surface of the boundary condition is satisfied

$$\hat{\mathbf{n}} \times (\mathbf{E}^{inc}(\mathbf{r}) + \mathbf{E}^{sc}(\mathbf{r})) = \mathbf{0}$$
$$r = a$$
(1)

We consider that the exterior $\Omega^+ = \mathbb{R}^3 / \Omega^-$ of the scatterer is homogeneous chiral media with fixed chirality β, dielectric constant ε and magnetic permeability μ.

A spherical incident electromagnetic wave ($\mathbf{E}_{\mathbf{r_0}}^{inc}(\mathbf{r})$, $\mathbf{H}_{\mathbf{r_0}}^{inc}(\mathbf{r})$) with time harmonic dependence in accordance with the Bohren transformation, analyzed in spherical Beltrami fields $\mathbf{Q}_{L,r_0}^{inc}(\mathbf{r})$ and $\mathbf{Q}_{R,r_0}^{inc}(\mathbf{r})$ as follows:

$$\begin{cases} \mathbf{E}_{\mathbf{r_0}}^{inc}(\mathbf{r}) = \mathbf{Q}_{L,r_0}^{inc}(\mathbf{r}) + \mathbf{Q}_{R,r_0}^{inc}(\mathbf{r}) \\ \mathbf{H}_{\mathbf{r_0}}^{inc}(\mathbf{r}) = \dfrac{1}{i\eta} (\mathbf{Q}_{L,r_0}^{inc}(\mathbf{r}) - \mathbf{Q}_{R,r_0}^{inc}(\mathbf{r})) \end{cases}$$
(2)

where $\eta = (\mu/\varepsilon)^{1/2}$ is the intrinsic impedance of the chiral medium. The Beltrami fields satisfy the equations, [5], [6],

$$\begin{cases} \nabla \times \mathbf{Q}_{L,r_0}(\mathbf{r}) = \gamma_L \mathbf{Q}_{L,r_0}(\mathbf{r}) \\ \nabla \times \mathbf{Q}_{R,r_0}(\mathbf{r}) = -\gamma_R \mathbf{Q}_{L,r_0}(\mathbf{r}) \end{cases}$$
(3)

where γ_L, γ_R are wave numbers for Beltrami fields and are given by,

$$\gamma_L = \frac{k}{1 - k\beta}, \gamma_R = \frac{k}{1 + k\beta}$$
(4)

With $k = \omega(\varepsilon\mu)^{1/2}$, ω being the angular frequency. The indices L and R denote the LCP and RCP fields respectively. The spherical incident Beltrami fields with suitable normalization have the form, as defined in the following works: [4] (issue 2, p. 9, relations (4), (5)), and [3].

If $\mathbf{Q}_{L,r_0}^{inc}(\mathbf{r})$ and $\mathbf{Q}_{R,r_0}^{inc}(\mathbf{r})$ are Beltrami fields corresponding via transformation to Bohren in $\mathbf{E}_{r_0}^{inc}$ and $\mathbf{H}_{r_0}^{inc}$, then the scattering problem for the perfect conductor can be formulated in the following way: be found $\mathbf{Q}_{L,r_0}^{tot}(\mathbf{r})$, $\mathbf{Q}_{R,r_0}^{tot}(\mathbf{r})$, which belong to the space $C^1(\Omega^+) \cap C(\overline{\Omega^+})$, such that:

$$(i): \begin{cases} \nabla \times \mathbf{Q}_{L,r_0}^{tot}(\mathbf{r}) = \gamma_L \mathbf{Q}_{L,r_0}^{tot}(\mathbf{r}) \\ \nabla \times \mathbf{Q}_{R,r_0}^{tot}(\mathbf{r}) = -\gamma_R \mathbf{Q}_{R,r_0}^{tot}(\mathbf{r}) \end{cases}$$

$$\mathbf{r} \in \Omega^+$$

$$(ii): \hat{\mathbf{n}} \times \mathbf{Q}_{L,r_0}^{tot}(\mathbf{r}) = -\hat{\mathbf{n}} \times \mathbf{Q}_{R,r_0}^{tot}(\mathbf{r})$$

$$\mathbf{r} \in S = \partial\Omega^-$$ (5)

$$(iii): \begin{cases} \hat{\mathbf{r}} \times \mathbf{Q}_{L,r_0}^{sc}(\mathbf{r}) + i\mathbf{Q}_{L,r_0}^{sc}(\mathbf{r}) = o(\frac{1}{r}) \\ \hat{\mathbf{r}} \times \mathbf{Q}_{R,r_0}^{sc}(\mathbf{r}) - i\mathbf{Q}_{R,r_0}^{sc}(\mathbf{r}) = o(\frac{1}{r}) \end{cases}$$

$$r \to \infty$$

The limits on radiation conditions (5(iii)), are taken uniformly in all directions $\hat{\mathbf{r}} \in S^2$, where S^2 is the unit sphere in $\mathbb{R}^3$ and $\hat{\mathbf{n}}$ is the outward normal unit vector perpendicular to the surface on the scatterer.

The incident electromagnetic wave $(\mathbf{E}_{r_0}^{inc}(\mathbf{r}), \mathbf{H}_{r_0}^{inc}(\mathbf{r}))$ on the scatterer Ω^- generates the corresponding scattered field $(\mathbf{E}_{r_0}^{sc}(\mathbf{r}), \mathbf{H}_{r_0}^{sc}(\mathbf{r}))$. The scattered electric field will be depended on the polarizations $\hat{\mathbf{p}}_L$, $\hat{\mathbf{p}}_R$, (see [4] (issue 2, p. 9, relation (6)) and will have the decomposition

$$\mathbf{E}_{r_0}^{sc}(\mathbf{r} \,|\, \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) = \mathbf{Q}_{L,r_0}^{sc}(\mathbf{r} \,|\, \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) +$$
$$+ \mathbf{Q}_{R,r_0}^{sc}(\mathbf{r} \,|\, \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R)$$ (6)

Where $\mathbf{Q}_{L,r_0}^{sc}(\mathbf{r} \,|\, \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R)$ and $\mathbf{Q}_{R,r_0}^{sc}(\mathbf{r} \,|\, \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R)$ are the corresponding scattered Beltrami fields which have the following behavior, when $r \to \infty$, [5], [7],

$$\mathbf{Q}_{A,r_0}^{sc}(\mathbf{r} \,|\, \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) = h_0(\gamma_A r) \cdot$$
$$\cdot \mathbf{g}_{A,r_0}(\hat{\mathbf{r}} \,|\, \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) + O(\frac{1}{r^2})$$ (7)

with $A = L, R$. The functions $\mathbf{g}_{L,r_0}$ and $\mathbf{g}_{R,r_0}$ are the LCP and RCP far-field patterns respectively, which are defined by the following relationship [7],

$$\mathbf{g}_{A,r_0}(\hat{\mathbf{r}} \,|\, \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) =$$
$$\int_S \hat{\mathbf{n}} \times [\gamma_A \nabla \times \mathbf{E}_{r_0}^{sc}(\mathbf{r}' \,|\, \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) -$$ (8)
$$- \varpi_A \gamma^2 \mathbf{E}_{r_0}^{sc}(\mathbf{r}' \,|\, \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) e^{-i\gamma_A \mathbf{r} \cdot \mathbf{r}'} ds(\mathbf{r}')$$

With $\varpi_A = \begin{cases} -1, A = L \\ 1, A = R \end{cases}$, $\gamma^2 = \gamma_L \gamma_R$.

Near-field Inverse problem

Using the exact solution found in the paper [4] (issue 2, pp. 10-12) and considering appropriate asymptotic forms of the Bessel and Hankel functions in low frequencies, i.e. $|y_A a| \ll 1$, calculate asymptotic expressions of the coefficients a_n^L, a_n^R, b_n^L and b_n^R. Specifically we have [3],

$$a_n^L \sim \frac{1 + \beta k}{2 i \zeta_n^2 (2n + 1)} (y_L a)^{2n + 1}$$

$$y_L a \to 0 \qquad (9)$$

$$a_n^R \sim -\frac{i}{2n\zeta_n^2} \frac{(1 - \beta k)^{n + 2}}{(1 + \beta k)^{n + 1}} (y_L a)^{2n + 1}$$

or

$$b_n^L \sim -\frac{i}{2n\zeta_n^2} \frac{(1 + \beta k)^{n + 2}}{(1 - \beta k)^{n + 1}} (y_R a)^{2n + 1}$$

$$y_R a \to 0 \qquad (10)$$

$$b_n^R \sim -\frac{i(1 - \beta k)}{2\zeta_n^2 (2n + 1)} (y_R a)^{2n + 1}$$

Where

$$\zeta_n = 1 \cdot 3 \cdot 5 \cdots (2n - 1) = (2n)! / (2^n n!)$$

In the inverse problem we calculate the scattered Near-field in the source $\mathbf{r} = \mathbf{r_0}$. By the relation (21), in work [4], with $\mathbf{r} = \mathbf{r_0}$ and for LCP incidence, we obtain [3],

$$\mathbf{E}_{\mathbf{r_0}}^{sc}(\mathbf{r_0} \,|\, \hat{\mathbf{p}}_L) =$$

$$= \sum_{n=1}^{\infty} \frac{n(n + 1)}{2} B_n^L a_n^L \{ (h_n(y_L r_0)\hat{\mathbf{x}} + \tilde{h}_n(y_L r_0)\hat{\boldsymbol{\psi}}) + i(\tilde{h}_n(y_L r_0)\hat{\mathbf{x}} - h_n(y_L r_0)\hat{\boldsymbol{\psi}}) \} + \quad (11)$$

$$+ \sum_{n=1}^{\infty} \frac{n(n + 1)}{2} B_n^L a_n^R \{ (h_n(y_R r_0)\hat{\mathbf{x}} - \tilde{h}_n(y_R r_0)\hat{\boldsymbol{\psi}}) + i(-\tilde{h}_n(y_R r_0)\hat{\mathbf{x}} - h_n(y_R r_0)\hat{\boldsymbol{\psi}}) \}$$

Using the asymptotic relations:

$$\frac{h_n(y_A r_0)}{h_0(y_A r_0)} \sim \frac{\zeta_n}{(y_A r_0)^n}$$

$$\frac{\tilde{h}_n(y_A r_0)}{h_0(y_A r_0)} \sim \frac{-n\zeta_n}{(y_A r_0)^{n + 1}} \qquad (12)$$

$$\tilde{h}_n(y_A a) \sim -\frac{n\zeta_n}{i(y_A a)^{n + 2}}$$

with $y_A r_0 \to 0$, $A = L, R$ and the series,

$$\begin{cases} \displaystyle\sum_{n=1}^{\infty} \tau^{2n + 1} = \frac{\tau^3}{1 - \tau^2} \\[4mm] \displaystyle\sum_{n=1}^{\infty} n^2 \tau^{2n + 1} = \frac{\tau^3(1 + \tau^2)}{(1 - \tau^2)^3} \end{cases} \qquad (13)$$

$$\sum_{n=1}^{\infty} (2n + 1)\, \tau^{2n + 1} = \frac{\tau(3\tau^2 - \tau^4)}{(1 - \tau^2)^2} \quad (14)$$

$$\sum_{n=1}^{\infty} n(2n+1)\tau^{2n+1} = \frac{\tau(3\tau^2 + \tau^4)}{(1-\tau^2)^3} \quad (15)$$

$$\sum_{n=1}^{\infty} \frac{2n+1}{n} \tau^{2n+1} = \qquad (16)$$

$$= \frac{\tau[\tau^2 \ln(1-\tau^2) + 2\tau^2 - \ln(1-\tau^2)]}{\tau^2 - 1}$$

and because of the relationships:

$$\frac{\gamma_R}{\gamma_L} = \frac{1-\beta k}{1+\beta k}, \quad 0 < \tau < 1,$$

where $\tau = a/r_0$, we obtain from (11) [3],

$$\mathbf{E}_{\mathbf{r}_0}^{sc}(\mathbf{r}_0 \mid \hat{\mathbf{p}}_L) \sim$$
$$\sim \frac{(1+\beta k)}{2(\gamma_L a)^2} \frac{\tau^5}{(1-\tau^2)^3} \cdot \hat{\mathbf{p}}_L \qquad (17)$$

Therefore the measure of the scattered field $\mathbf{E}_{\mathbf{r}_0}^{sc}(\mathbf{r}_0 \mid \hat{\mathbf{p}}_L)$ is

$$\left| \mathbf{E}_{\mathbf{r}_0}^{sc}(\mathbf{r}_0 \mid \hat{\mathbf{p}}_L) \right| \sim \frac{(1+\beta k)}{2(\gamma_L a)^2} \frac{\tau^5}{(1-\tau^2)^3} \quad (18)$$
$$\gamma_L a \to 0$$

Similarly for the RCP incidence, we have that

$$\left| \mathbf{E}_{\mathbf{r}_0}^{sc}(\mathbf{r}_0 \mid \hat{\mathbf{p}}_R) \right| \sim \frac{(1-\beta k)}{2(\gamma_R a)^2} \frac{\tau^5}{(1-\tau^2)^3} \quad (19)$$
$$\gamma_R a \to 0$$

Finally, based on the relationships (18) and (19), we conclude that

$$\left| \mathbf{E}_{A,\mathbf{r}_0}^{sc}(\mathbf{r}_0 \mid \hat{\mathbf{p}}_A) \right| \sim$$
$$\sim \frac{(1-\varpi_A \beta k)}{2(\gamma_A a)^2} \frac{\tau^5}{(1-\tau^2)^3} \qquad (20)$$
$$\gamma_A a \to 0, \quad A = L, R$$

Choose a Cartesian coordinate system $Ox\psi z$, and five point-source locations, namely $O(0,0,0)$, $A_1(l,0,0)$, $A_2(0, l, 0)$, and $A_4(0,0,2l)$, which are at (unknown) distances r_0, r_1, r_2, r_3 and r_4, respectively from the sphere's center K. The parameter l is a chosen fixed length. The sizes of the resulting five measurements $\left| \mathbf{E}_{A,\mathbf{r}_j}^{sc}(\mathbf{r}_j \mid \hat{\mathbf{p}}_A) \right|$ are

$$M_j = \frac{2(\gamma_L \gamma_R)l}{(1-\varpi_A \beta k)} \left| \mathbf{E}_{A,\mathbf{r}_j}^{sc}(\mathbf{r}_j \mid \hat{\mathbf{p}}_A) \right| \qquad (21)$$

and

$$\rho_j = \frac{r_j}{l}, \quad b = \sqrt{\frac{a}{l}} \qquad (22)$$

Where $j = 0, 1, 2, 3, 4$. Therefore we have the following five measurements

$$M_j = \frac{b^5 \rho_j}{(\rho_j^2 - b^4)^3} \qquad (23)$$

with $\rho_j > b^2 > 0$. Applying the law of cosines in triangle KOA_4 and using that

$$r_j = \rho_j l \qquad (24)$$

we obtain

$$\rho_4 = 2 + 2\rho_3 - \rho_0 \qquad (25)$$

Because of (24), we take

$$\rho_j^6 - 3\rho_j^4 b^4 + 3\rho_j^2 b^8 -$$

$$- b^{12} - b^5 \frac{\rho_j}{M_j} = 0 \qquad (26)$$

Thus in equation (26) identified the ρ, with $j = 0, 1, 2, 3, 4$. If the radius a of the sphere is known, solve the system of six algebraic equations (25) and (26). Also if you consider that the radius

a is too small so that $a \ll l$, then the relationship (23) shows [3],

$$M_j = \frac{b^5}{\rho_j^5} \qquad (27)$$

so we have the following system of equations

$$\begin{cases} \rho_4 = 2 + 2\rho_3 - \rho_0 \\ \rho_j^5 M_j - b^5 = 0 \end{cases} \qquad (28)$$

where $j = 0, 1, 2, 3, 4$.

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