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Interfaces in High Voltage Engineering: A Most Important Question for Conventional Solid Insulating Materials as well as for Nanocomposite Polymers

M.G. Danikas, R. Sarathi *

Power Systems Laboratory, Department of Electrical and
Computer Engineering, Democritus University of Thrace,
Xanthi, Greece [1]
Department of Electrical Engineering,
Indian Institute of Technology Madras, Chennai, India [2]

Abstract

Interfaces consist a most important part of conventional insulating systems at high voltages. They are considered to be problem areas which have to be dealt with. Numerous publications have contributed in rendering the mechanisms of interfaces understandable. On the other hand, interfaces in nanocomposite polymers seem to function in an entirely different manner from that in conventional insulating systems. The present paper reviews past work on both the conventional insulations and in nanocomposites. Differences regarding the interfaces are mentioned and discussed. Whereas interfaces in conventional insulating systems are to be avoided, interfaces in nanocomposite polymers seem to be desirable – at least – up to a certain percentage of nanoparticles in the base polymer. Although things are better understood in conventional insulating materials, more work has to be performed in order to clarify several aspects, such as space charges and electrical trees emanating from enclosed cavities. Needless to say that much more work has to be done in nanocomposites w.r.t. their modeling and possible explanations of the surprising performance of interfaces, a performance that deviates strongly from the performance in classical insulating materials.

Keywords

Breakdown, breakdown strength, conventional insulating materials/systems, nanocomposite polymers, partial discharges, electrical treeing/trees

Introduction

With some sort of exaggeration, a well known professor said once that "the problems of high voltage insulations are problems of interfaces" [1]. Interfaces result when there are two different insulating materials next to each other or when an insulating material meets a conductor. An interface may become, e.g., the source of partial discharges or even the cause for a complete breakdown of an insulating system. When two insulating materials of different dielectric constants have a common interface, then the material with the lowest dielectric constant will undergo the more intense electric stressing [2]. Depending also on other parameters, such stressing may result in a gradual deterioration of the insulating system and consequently in a complete breakdown [3].

In this paper, the questions raised by the existence of interfaces in high-voltage insulating systems are discussed. Interfaces play an important role in determining the robustness of an insulating system, when conventional insulating materials are used. Interfaces play also a vital role in insulating systems with nanocomposite polymers. Their functioning, however, is of another nature. This short review is by no means exhaustive since the topic of interfaces is a vast one. It is the aim of the authors to give the gist of the problems and questions the researchers may face and to offer some comments. In the context of this paper, the terms "insulating material" and "dielectric material" or simply "dielectric" are sometimes interchanged meaning the same thing.

Interfaces in Conventional Insulating Materials

Dielectric breakdown in insulating materials depends on electrode configuration, insulating material thickness, electrode materials, presence of microcavities, temperature, pressure, nature and morphology of material under test, type of applied voltage, damage path (surface or volume) [4]. Various dielectric breakdown theories have been put forward [5]-[13]. No matter whether the proposed theory was based on cumulative impact ionization by electrons – creating thus positive space charges which distort the field distribution and weaken the dielectric [5] –, on the notion of "intrinsic breakdown" – according to which a large number of electrons trapped in energy levels due to lattice imperfections can transfer energy to the lattice vibrations [6]-[8], on the "40 generations avalanche theory" [9], on the importance of space charges which modify the local electric field value [10], on the right assumption that the breakdown is a property of the dielectric material plus its electrode system [11] or on the theory based on the ionizing electrons and the hole traps [12], [13], the fact remains that all the above mentioned phenomena result from electric field intensifications, i.e. from either electrode imperfections or mismatch of dielectric materials. This brings us to the point mentioned in the Introduction of the present paper: that interfaces may create the conditions which may cause electric field intensifications.

The subject of partial discharges (PD) which may ensue because of electric field intensifications and/or because of gas (or foreign particle) inclusions in a solid dielectric material, has been studied by Mason in his fundamental publications [14]-[18]. Having in mind all the above, it is fitting to say that interfaces – created either by a mismatch of dielectric materials or because of intrusion of foreign particles and/or air cavities in the insulating material under question –, are the problem areas of an insulating system.

As was pointed out quite early [19], interfaces play a most significant role in the discharge and breakdown processes: even if an insulation does not contain any cavities, at a sufficient stress "some event" releases gas with the subsequent formation of a cavity. The cavity is occupied by a gas discharge which increases the rate of gas formation with the subsequent growth of both the cavity size and the discharge intensity. The importance of the differing nature of interfaces was also stressed in another publication, where it was pointed out that damage in internal cavities in polyethylene is little compared to the electrode adjacent cavities of the same dimensions and tested under the same experimental conditions [20]. It is evident that in [20], interfaces between polyethylene and gas were compared with interfaces between polyethylene and electrodes, and the latter were found to be more dangerous and deleterious to the insulating material. On the same lines, Kreuger showed that with PVC-insulated cables, the number of discharges increased with increasing electric stress in the dielectric [21]. In yet another paper, it was indicated that the nature of internal discharges was greatly affected by the assembly of the electrode system and the adhesion of the insulating tapes [22]. Discharges always start in the electrically weaker insulating material, as was commented in previous works [23]-[25].

Needless to say that phenomena related to PD, such as electrical treeing, are also closely connected to the mismatch of the dielectric constants of insulating materials and/or to the existence of gas cavities in their volume. Earlier papers indicated that the treeing phenomenon in polyethylene cables started from both inner and outer surfaces and also from solid particles and fibres [26]. Pioneering work performed with 15 kV and 22 kV polyethylene insulated cables reported that trees originated from contaminants and cavities, the tendency for tree initiation from a contaminant being probably more affected by the contaminant material than by the size, location or shape of the contaminant [27]. The importance of enclosed cavities in the initiation of trees was also reported more recently [28]. According to Ieda [29], tree propagation can be induced by internal gas discharge in the tree. It is to be bore in mind that in numerous publications dealing with experimental work, the electrode arrangement that was used was a needle-plane electrode arrangement, indicating again that an electrode arrangement was chosen, with pronounced interfaces, in order to study the treeing phenomenon [30]-[33].

Interfaces which may play a role in determining the breakdown strength of an insulating material need not be only interfaces between insulating material and metal or between insulating material and gas cavity or contaminant. Differing phases may play also a role, as was noted in [34], where the interfacial domain of crystalline and amorphous phases may determine the various properties of semi-crystalline polymers, such as biaxially orientated polypropylene (BOPP). Stressing the importance of interfaces and experimenting with cross-linked polyethylene (XLPE), McKean showed that a considerable improvement in cable breakdown can be achieved by impregnation with silicone oil or diethyleneglycol. Such liquids can impregnate gas microspaces in the main insulation and thus result in an increase of the breakdown strength [35]. Similar observations were also reported with polypropylene and polyethylene impregnated with suitable dipolar liquids [36].

Interfacial breakdown was studied with various electrode geometries and insulating systems consisting of paper typical for transformers and transformer oil [37]. It was reported that interfacial breakdown will occur if the paper is not carefully dried or if many gaseous microporosities are left in or on the paper. In [37], however, it was also noticed that using a carefully prepared paper-oil interface structure, the breakdown does not necessarily take place at the interface. Similar observations were made more recently by using silicone rubber interfaces, where both perpendicular and parallel to the applied electric field were investigated [38].

Conventional paper-oil cable insulation was studied quite early and the problems of interfaces were noted [39]. Alternative insulating systems, based mainly on polymeric materials, were proposed with considerable commercial success [40] - [43]. Modern cables with solid polymeric insulation did not avoid the problems of interfaces, namely those of extrusions of semi-conducting sheaths with the main insulation or the inclusion of microcavities and/or impurities [44], [45]. Extruded cable insulation exposed to wet conditions suffered from electrochemical treeing and impurities greatly deteriorated its electrical performance [46]. Moreover, operating electrical stresses may also cause premature insulation failure in 15 kV polyethylene cables, if combined with unfavorable interface profiles and moisture [47]. Interfaces between polyethylene and small contaminants or microcavities may cause bow-tie trees in polyethylene cables [48].

Interfaces either perpendicular to the applied electric field or parallel to it or at an angle with it were dealt with in [49], where it was noted that such a variety of interfaces may be encountered in applications, such as capacitors, cables and in transformer windings. Composite insulating systems must preserve low dielectric losses. Higher dielectric losses may imply high ionic concentration in a solid/liquid insulating system, i.e. high ionic concentration in the liquid component of such a system [50].

Before concluding this section, it is fitting to mention the composite insulating systems of electrical machines, which consist mainly of epoxy resin and mica sheets. Previous work done in this context indicated that electrical treeing propagates through the epoxy resin and generally stops at the mica sheets, as mica is harder and electrically stronger than epoxy resin. The importance of such interfaces was reported before using a needle-plane electrode arrangement [51], where experiments were carried out without and with a mica sheet inserted in epoxy resin (Figs. 1 and 2). Evidently electrical trees were propagating more easily in the case of absence of the mica sheet and with much more difficulty with mica sheet.

Simulation work done recently showed that mica sheets prevent electrical trees from reaching the opposite electrode [52]. The purpose of mentioning experimental results regarding the composite system of epoxy resin/mica sheets is to show that the electrical trees propagate through the electrically weaker insulating medium. The simulation results indicate that even the slightest variations of dielectric constant may cause the electrical tree growth. In other words, the simulation data indicate that local fluctuations of dielectric constant imply local – even microscopically minute – formations of interfaces, which in turn may mean local field intensifications, encouraging thus the growth of electrical trees (Figs. 3 and 4). Such observations w.r.t. the local variations of dielectric constant have also been reported for polyethylene [53], [54].

It is evident from all the above that interfaces in classical insulating systems seem to cause problems (possible dielectric constant mismatch, PD, treeing phenomena and ultimately risk of ultimate insulation failure). Due attention should be paid in choosing the insulating materials for specific applications and to the construction of the composite insulating system. Too many things depend on the quality of the construction of the interfaces [55]-[58], too many things that cannot be ignored. Interfaces in traditional insulating systems are considered as the weak aspects of such systems. Keeping this in mind and without exaggerating, it is not far from the truth if we state that an insulating system is as good as its weakest interfaces.

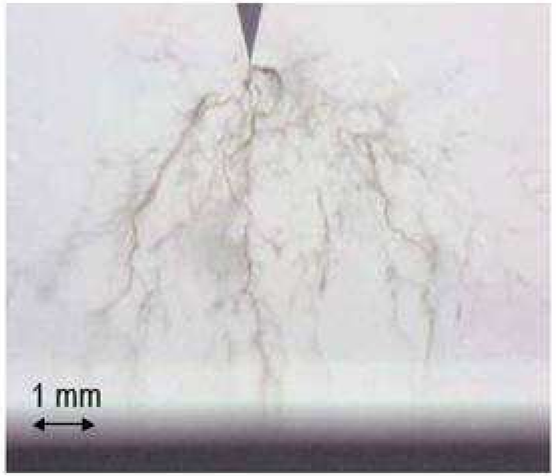

Fig. 1: Electrical tree propagation without the presence of mica sheet (applied voltage 28 kVrms, 50 Hz) (after [51])

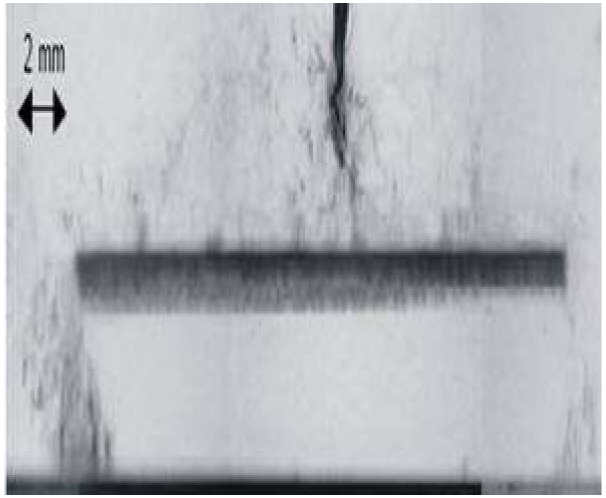

Fig. 2: Electrical tree propagation with the presence of mica sheet. The mica sheet increases the propagation time of the tree (applied voltage 28 kVrms, 50 Hz) (after [51])

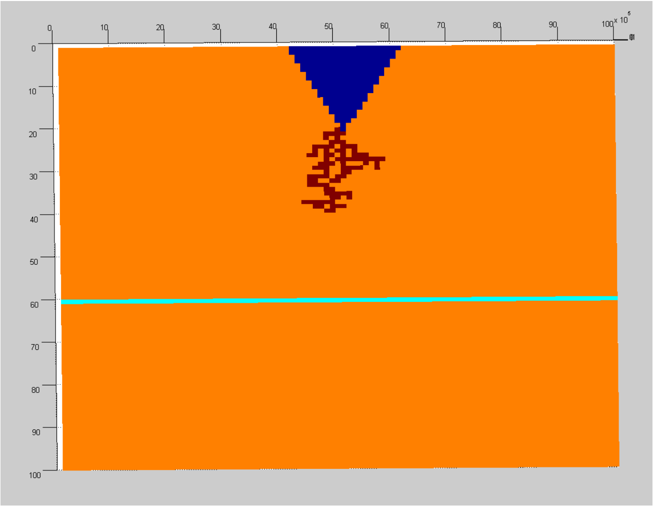

Fig. 3: Simulated electrical tree propagation with one mica sheet. Needle-plane electrode arrangement used

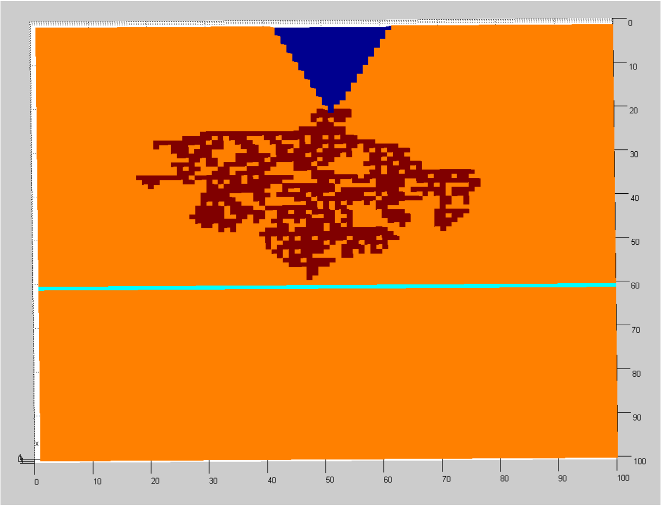

Fig. 4: Further expansion of electrical trees. Electrical trees stop at mica sheet. Needle-plane electrode arrangement used

It goes without say that this short review regarding classical interfaces does not by any means cover the whole subject and variety of solid insulating materials (for example, no mention in this paper was offered about the interfaces in outdoor polymeric insulators [59], [60] or in indoor polymeric insulators [61]). Both in the libraries and in the Internet, the interested reader may find practically tens of thousands of publications referring to the questions and problem areas of the solid insulating materials and insulating systems. What this short review tried to do is to show that interfaces, electric field intensifications, pre - breakdown phenomena (such as PD and electrical trees) and breakdown mechanisms are all interwoven and interrelated. Having said that, the next question related to interfaces, is whether they play the same detrimental role in the new generation of insulating materials, the nanocomposite polymers. This is to be examined in the following section.

Interfaces in Nanocomposite Polymers

More than twenty years ago, nanocomposite polymers came to our lives [62]. The first application of nanocomposites appeared in 1990, when Toyota Motor Corporation introduced nanocomposite nylon in their car industry [63]. Since that year, many car industries introduced the use of nanocomposite polymers. Use of nanocomposite polymers was noted in other industries, such as in the optics and the electronics industries as well as in the food industry. A seminal paper by Lewis gave the impetus for research also in the insulation branch [64].

For the electrical insulation, nanocomposite polymers are defined as conventional polymers in which particles smaller than 100 nm are added and dispersed in such a way that at the end one gets a homogeneous mixture [63]. The addition of such nanoparticles (the term "nanofillers" is also widely used) is being done in very small quantities, usually less than 10 wt%. Nanocomposite polymers consist of three components:

a) the base polymer (or polymer matrix),
b) the nanoparticles (or nanofillers) and
c) the interaction zone (or interface zone) between the matrix and the nanoparticles [63].

Regarding the polymers used, these may be either thermoplastics, thermosettings or elastomers. Nanoparticles may be classified w.r.t. their dimensions, and they can be distinguished as

1) mono-dimensional (i.e. extremely thin),
2) two-dimensional (nanotubes) and
3) three-dimensional (inorganic oxides).

The most usual nanoparticles for the purposes of electrical insulation are

1) silica nanoparticles SiO_2,
2) montmorillonite nanoparticles (layered silica),
3) metallic oxides such as Al_2O_3, TiO_2, MgO and ZnO and
4) carbon nanotubes.

Nanocomposite polymers can be obtained in two types of structures, namely,

(i) intercalated nanocomposites (formed when there is limited inclusion of polymer chain between the clay layers with a corresponding small increase in the interlayer spacing of a few nanometers and
(ii) exfoliated nanocomposites (formed when the clay layers are well separated from one another and individually dispersed in the continuous polymer matrix [65], [66].

As mentioned above, nanoparticles are added and dispersed in relatively small quantities in the base polymer (usually no more than 10 wt%). Since nanoparticles are smaller than microparticles (smaller by three orders of magnitude), their interaction with the surrounding polymer matrix is much greater [62]. The so-called interaction zone is the main factor contributing in the improvement of the properties of the base polymer [67]. In the case of addition of nanoparticles into a polymer, the interfaces are far more numerous and far larger than in the case of microparticles. As the size of the added particles is reduced, the interface becomes larger and larger. The distance between the nanoparticles is also extremely small. It seems that interfaces determine to a great extent the properties of nanocomposite polymers.

The size of nanoparticles and the distance between them is of the order of magnitude on nanometers. Such particles may interact with the polymer matrix both physically and chemically in the nanometer scale. This has as consequence the appearance of properties that are somehow different from those we already know in a more macroscopic scale [68]. In contradistinction to the interfaces in classical insulating materials, and also to what we know from classical high voltage textbooks, the improved insulating properties of the nanocomposite polymers are due to

a) the large surface area of nanoparticles, which creates a large interaction zone,
b) the changes in polymer morphology because of the large interaction zone,
c) the changes in the space charge distribution and
d) a dispersion mechanism [69].

Both the size of the nanoparticles and the chemical properties of their surface play an important role in determining the electrical, thermal and mechanical properties of nanocomposites. Needless to say that the chemical compatibility between the introduced nanoparticles and the polymer matrix is of paramount importance for the general properties of the nanocomposite [70].

One of the most significant characteristics of nanocomposite polymers is the increase of their breakdown strength as the size of the added nanoparticles tends to extremely small values. This increase is not in agreement with the conventional wisdom, which suggests that as the number of interfaces increases, the breakdown strength decreases dramatically [2], [3]. Nanocomposite polymers seem not to agree with what we already know for classical insulating materials or systems [71]. Differences in breakdown strength between conventional epoxy resin and epoxy resin with nanoparticles was reported in [72]. Such observations were also noted later, when six different materials based on epoxy resin with various with and/or micro- and nanoparticles of alumina/silica were tested. It was shown that epoxy resin with nanosilica particles was the most suitable to obtain high values of breakdown strength [73].

Addition of the percentage of nanoparticles to epoxy resin up to a certain level favors the increase of breakdown strength both with a.c. and d.c. voltage, as was noticed in [74]. Why nanoparticles act in such a favorable way, despite the numerous interfaces they create? The increase of the breakdown strength may be due to

(i) the increase of the surface area of the interfaces, which somehow alters the behavior of the polymer,
(ii) the changes of space charge distribution inside the insulation structure,
(iii) the dispersion mechanism, and
(iv) the changing properties of the insulating material, more specifically its volume resistivity, its tanδ and its dielectric constant.

It is possible that the electrons moving in such a nanocomposite polymer, loose their kinetic energy because of the nanoparticles. Since the distances between the nanoparticles are extremely small, the electrons cannot acquire enough speed so that they can contribute to the breakdown process. Consequently, epoxy resin with nanoparticles presents a higher breakdown strength than conventional epoxy resin [74].

Normally the introduction of particles in polymeric materials has as result the introduction of defects and subsequently the worsening of its electrical properties. Nanocomposite polymers seem not to obey the above rule, as the mechanisms of conductivity during the breakdown process are influenced from the applied electric field, the dielectric constant of the nanoparticles and their number. The combined effect of these factors is difficult to fully understand at this stage and we need more work [71]. Similar results were obtained with epoxy resin with nanoparticles of TiO_2 as it presented a much higher breakdown strength than conventional epoxy resin [75].

On the other hand, electrical treeing propagation was found to be easier in conventional polymers than in nanocomposite polymers [76], [77]. Even a small wt% addition of nanoparticles affects in a positive way the electrical treeing resistance of the nanocomposite polymer. It seems that electrical tree propagation paths go through the base polymer and around the nanoparticles (experimental evidence for this was presented in SEM photographs published in [76]). Consequently, the more the nanoparticles in a polymer, the more difficult the formation of treeing paths. It seems that nanoparticles act as extremely small barriers, thus preventing the easy growth of electrical trees. Electrical trees propagate through the base polymer (in other words through the polymer matrix) and not through the nanoparticles. In some cases the electrical trees stop at the nanoparticles and they do not progress any further. Such observations were made in simulation studies recently [78]-[80].

Further research showed that a small percentage introduction of nanoparticles into a conventional polymer may increase its resistance to electrical treeing [81]. It is interesting to note that nanoparticles may function as barriers preventing the tree growth even in minute quantities [82]. Earlier work indicated that as soon as the electrical tree tou-ches the nanoparticle, the physico-chemical properties of it are such that very high energies are required in order to cause its deterioration [83]. Although the latter paper is an old one, it may give a clue as to why nanoparticles act as elementary barriers and why they prevent (or they delay) electrical treeing. More recently, a similar argument was given by some Japanese researchers [84].

Loading (i.e. the percentage of included nanoparticles expressed in wt%) plays also a vital role in determining the resistance to electrical treeing. More loading (i.e. more nanoparticles, that is more interfaces) implies a better resistance to treeing. This is probably because trees interact with many more nanoparticles and this delays their growth [85]. Another possible explanation was offered some years ago, where the authors proposed that in front of a tree a damage process zone is formed in a conventional polymer. Such a zone cannot progress easily when it meets nanoparticles [86]. Simulation data indicated that loading affects the tree growth. The nanoparticle size plays also a role in delaying tree propagation, smaller nanoparticles offer a better tree resistance than the larger ones [79]. In Figs. 5-8 simulation results regarding the loading of nanoparticles as well as the size of nanoparticles are shown. It is evident that more loading makes tree growth more difficult. It goes also without say that smaller nanoparticles offer a better resistance to tree propagation.

There is no need to emphasize that there is also in the field of nanocomposite polymers a vast body of technical literature, too vast to be mentioned here in the context of this paper. From this short review it is obvious that interfaces in nanocomposites play an entirely different role from the one they play in classical insulating materials. Why is this so? This may be because the physics and/or chemistry somehow change in the nanoscopic world of such materials. Interfaces become highly desirable – at least up to a certain percentage of added nanoparticles. The surface area of the nanoparticles is huge if compared with that of microparticles for other high voltage applications.

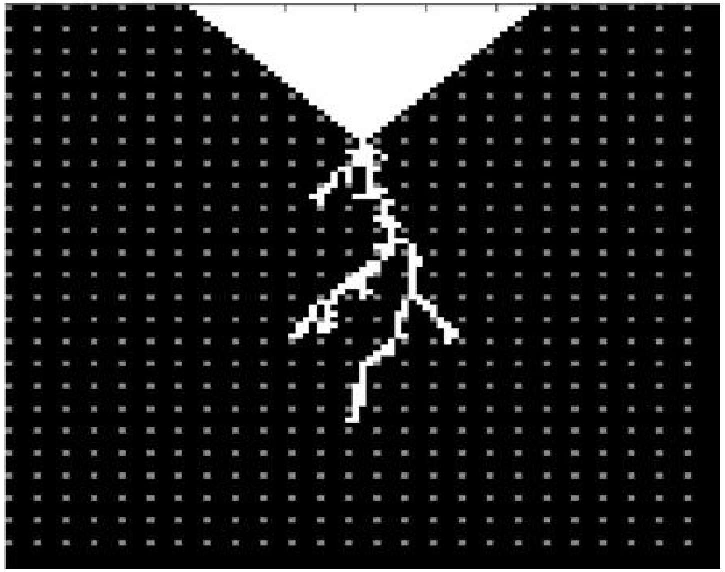

Fig. 5: Simulation with epoxy resin filled with TiO_2 nanoparticles (loading of 2 wt%, nanoparticle diameter 100 nm)

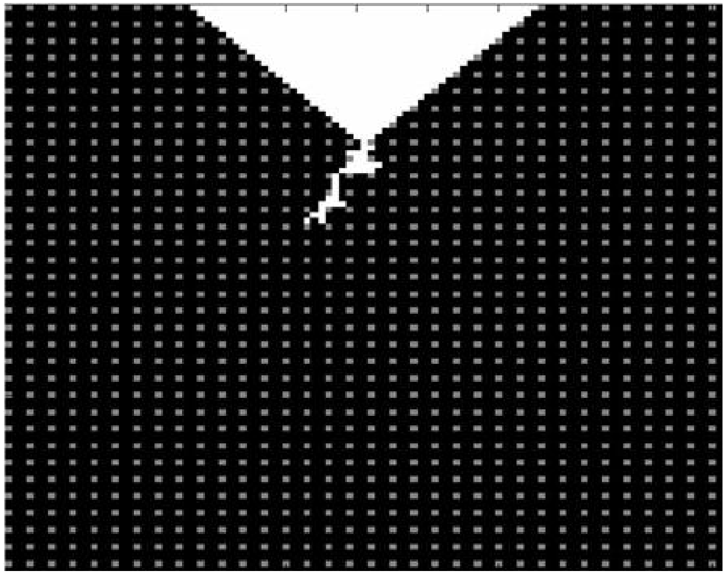

Fig. 6: Simulation with epoxy resin filled with TiO_2 nanoparticles (loading of 6 wt%, nanoparticle diameter 100 nm)

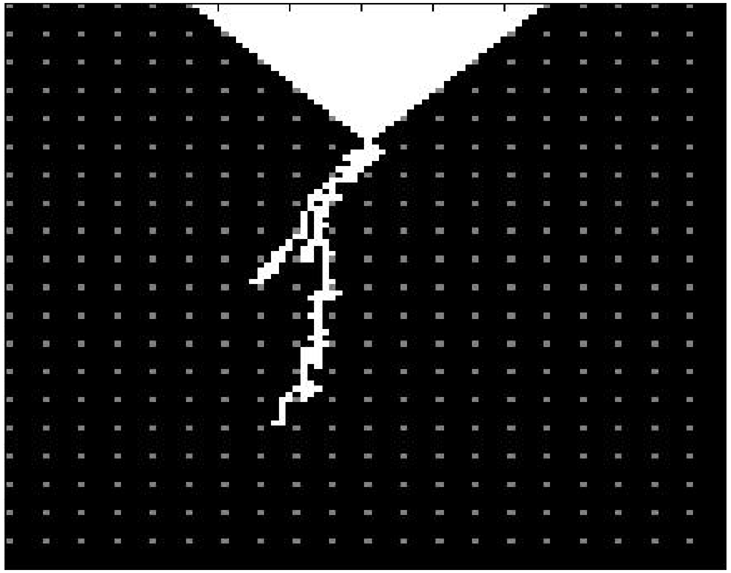

Fig. 7: Simulation with epoxy resin filled with TiO_2 nanoparticles (loading of 6 wt%, nanoparticle diameter 200 nm)

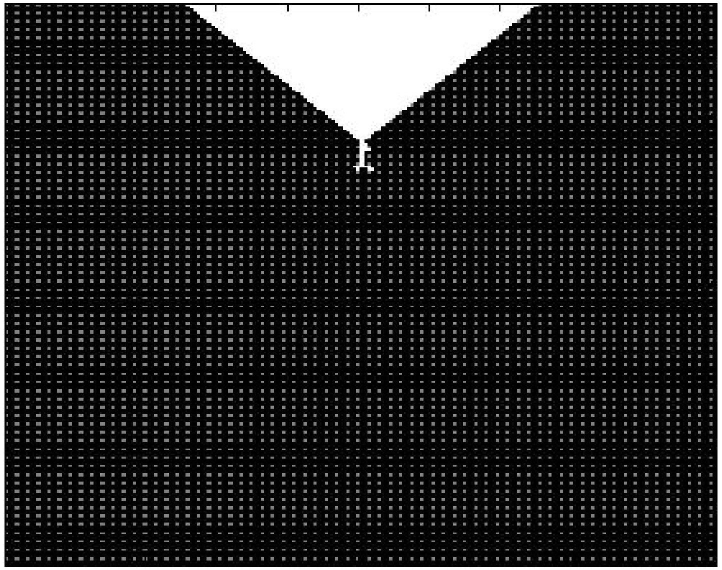

Fig. 8: Simulation with epoxy resin filled with TiO_2 nanoparticles (loading of 6 wt%, nanoparticle diameter 100 nm)

Models regarding the explanation of the functioning of nanocomposites were proposed as well as how the nanoparticles behave inside the base polymeric material [62], [87], [88]. Such models try to explain the higher breakdown strength values of nanocomposites and also their higher resistance to electrical treeing. The explanations seem to be plausible but more hard evidence is needed. Such evidence will be offered by many more photographs (SEM and TEM photos) showing in detail how the electrical trees circumvent the nanoparticles and how they propagate through the polymer matrix. Such photographs are very difficult to obtain.

Where all this leaves us? How can we understand in a unified way interfaces in both classical and nanocomposite insulating systems? Can the experimental results and simulations on electrical treeing with typical machine insulation ([52]) provide a hint also for a possible explanation in the nanoscopic world? A recent paper on interfaces posed some pertinent questions regarding classical and nanocomposite insulating materials [89]. From this paper it is obvious that although more questions are in need of an answer for the nanocomposites, the subject of interfaces in classical insulating systems is by no means finished. For example, charging of larger interfaces, such as found in cable joints and terminations, needs to be further explored regarding the mechanisms of space charges. Another aspect in need of further discussion is whether electrical trees may emanate from enclosed cavities in conventional polymers. The latter question has been partly answered in [28], [90] – where some experimental evidence was offered as to the possibility of electrical trees stemming from enclosed cavities – but further experimental data is needed.

One thing that should not be forgotten – and it is common to conventional as well as nanocomposite polymers – is that an insulating system to a great extent is as good as its interfaces. This means that, no matter whether we deal with conventional insulating materials or nanocomposites, preparation and construction in both cases has to be not only careful but meticulous.

A last remark on the literature presented here: the interested reader may find that the authors dwell perhaps too much in the older scientific literature. This is not done because they tend to ignore the more recent research: they simply would like to show that even in the old days, the problems were more or less the same. Moreover, they would also like to show that fundamental ideas – which are with us even today – on the various mechanisms in insulating materials came about quite early.

Conclusion

This review – by no means exhaustive – tackled the subject of the importance of interfaces both in conventional insulating materials and in nanocomposite polymers. Whereas interfaces are to be avoided in conventional materials, they seem to be a blessing in nanocomposites. Whereas in conventional materials they cause problems of compatibility and sometimes high field intensifications with all the bad consequences such intensifications entail (i.e. PD, trees), in nanocomposites they seem – up to a certain loading – to be desirable and they prevent (or the delay) tree growth. Whereas in conventional insulating systems the introduction of more interfaces seems to cause sometimes insurmountable problems, the introduction of interfaces (because of the nanoparticles' introduction to the polymer matrix) seems to alleviate electrical trees and to distribute more evenly electric fields and space charges.

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Spherical Beltrami Fields in Chiral Media: Reciprocity and General Theorems

Nikolaos M. Berketis *

Independent Researcher, Athens, Greece

Abstract

The Beltrami fields are extremely important for the description of time-harmonic electromagnetic fields in chiral media. This paper introduces the spherical Beltrami fields in chiral media, extending the corresponding known results for plane electromagnetic waves. Two key theorems are given for the scattering of spherical electromagnetic waves in chiral environment from a perfect conductor: the Reciprocity and the General Scattering theorems.

Keywords

Chiral media, spherical Beltrami fields, reciprocity relation, general scattering theorem

Introduction

The name "Beltrami field" comes from the fact that these fields $\mathbf{A}$ satisfy the equation type $\nabla \times \mathbf{A} = \alpha \mathbf{A}$, where $\alpha \neq 0$ is a reciprocal length [1]. The word chiral regards the concept of "chirality", derived from the Greek word hand. In geometry, a figure is chiral if it cannot be mapped to its mirror image by rotations and translations alone. Typical examples are human hands, snail shells, spirals and spirals in general.

The discoveries of natural optical activity in special materials have been known since the beginning of last century. Though optical activity has been considered in optics and in quantum mechanics for many years, its analysis within the framework of classical electromagnetic field theory arose much later. Recently, there has been a considerable interest in the study of scattering and diffraction by chiral medium. Chiral media are isotropic birefringent substances that respond to either electric or magnetic excitation with both electric and magnetic polarizations. Such media have been known since the end of the nineteenth century (e.g. the study of chirality by Pasteur) and find a wide range of applications from many sciences [1].

In general, the electromagnetic fields inside the chiral medium are governed by Maxwell equations together with Drude-Born-Fedorov equations in which the electric and magnetic fields are coupled [1]. The chiral medium is characterized by the electric permittivity ε, magnetic permeability μ and chirality measure β.

Scattering problems with incident waves have been studied: for spherical acoustic waves in the works [2]-[4], as well in the works [2], [5] for spherical electromagnetic waves in non-chiral electromagnetic waves and finally for spherical electromagnetic waves in chiral media in the works [6], [7].

In this work, we derive reciprocity relation and general scattering theorem, from spherical electromagnetic waves emanating from point sources in chiral media and scattered by a perfect conductor. Similar theorems are proved, analogously, for a chiral dielectric [6], [7].

As it is well known [1], in a homogeneous isotropic chiral medium the electromagnetic field is composed of left-circularly polarized (LCP) and right-circularly polarized (RCP) components which are propagated independently and with different phase speeds. When either a LCP or a RCP or a linear combination of LCP and RCP electromagnetic waves is incident upon a chiral scatterer then the scattered field is composed of both LCP and RCP components. So, using Bohren decomposition [1], [6], [7], the electromagnetic waves are expressed in terms of LCP and RCP Beltrami fields. The use of the Beltrami fields for problems of scattering in chiral material leads to a more simplified relationship.

This is due to the fact that the differential equations which satisfy Beltrami fields is of first order while the electric and magnetic field satisfy the Helmholtz modified equation which is of second-order. Moreover, the Beltrami equations and the conditions of radiation for the Beltrami fields are in effect separately for each one of them but on the scatterer surface, when applying the boundary conditions, both Beltrami fields are present. This fact makes easier to use these equations mainly in scattering problems where the behavior of the scattered field away from the scatterer is studied.

In the second section, we formulate the problem for spherical electromagnetic wave in a chiral medium. In the third section we formulate the problem of electromagnetic wave scattering from a perfect conductor as a function of the Beltrami fields using the Bohren transformation. The spherical Beltrami fields are defined in the fourth section. Finally, Reciprocity relation and the General Scattering theorem are given in fifth section.

Problem Formulation

Let Ω^- be a bounded and closed subset of $\mathbb{R}^3$ having a C^2-boundary S, i.e. $\partial\Omega^- = S$. The set Ω^- will be referred to as the scatterer. The exterior $\Omega^+ = \mathbb{R}^3/\Omega^-$ of the scatterer is an infinite isotropic homogeneous chiral medium with electric permittivity ε, magnetic permeability μ and chirality measure β.

The scatterer is filled with a isotropic chiral medium with corresponding physical parameters ε^-, μ^- and β^-. All the physical parameters are assumed to be real positive constants.

Let $(\mathbf{E}^{\mathrm{inc}}_{\mathbf{r}_0}, \mathbf{H}^{\mathrm{inc}}_{\mathbf{r}_0})$ be a time-Harmonic spherical electromagnetic wave, due a point source located at a point with position vector $\mathbf{r}_0$ with respect to an origin O in the vicinity of Ω^-.

This wave is incident upon the scatterer Ω^- and let $(\mathbf{E}^{\mathrm{sc}}_{\mathbf{r}_0}, \mathbf{H}^{\mathrm{sc}}_{\mathbf{r}_0})$ be the corresponding scattered field. Then the total electromagnetic field $(\mathbf{E}^t_{\mathbf{r}_0}, \mathbf{H}^t_{\mathbf{r}_0})$ in Ω^+ is given by

$$\begin{cases}\mathbf{E}^t_{\mathbf{r}_0}(\mathbf{r}) = \mathbf{E}^{inc}_{\mathbf{r}_0}(\mathbf{r}) + \mathbf{E}^{sc}_{\mathbf{r}_0}(\mathbf{r}) \\ \mathbf{H}^t_{\mathbf{r}_0}(\mathbf{r}) = \mathbf{H}^{inc}_{\mathbf{r}_0}(\mathbf{r}) + \mathbf{H}^{sc}_{\mathbf{r}_0}(\mathbf{r})\end{cases} \tag{1}$$

We assume that the scattered field satisfies the Silver-Müller radiation condition

$$\frac{1}{\eta}\mathbf{E}^{sc}_{\mathbf{r}_0}(\mathbf{r}) + \hat{\mathbf{r}} \times \mathbf{H}^{sc}_{\mathbf{r}_0}(\mathbf{r}) = o(r^{-1}), \quad r \to \infty \tag{2}$$

uniformly in all directions, $\hat{\mathbf{r}} \in S^2$, where S^2 is the unit sphere in $\mathbb{R}^3$, $r = |\boldsymbol{r}|$, $\hat{\mathbf{r}} = \boldsymbol{r}/r$ and $\eta = (\mu/\varepsilon)^{1/2}$ is the intrinsic impedance of the chiral medium in Ω^+. It is known [8], that (2) can be replaced by

$$\hat{\mathbf{r}} \times \mathbf{E}^{sc}_{\mathbf{r}_0}(\mathbf{r}) - \eta\mathbf{H}^{sc}_{\mathbf{r}_0}(\mathbf{r}) = o(r^{-1}), \quad r \to \infty \tag{3}$$

uniformly in all directions, $\hat{\mathbf{r}} \in S^2$. The total electric field $\mathbf{E}^{tot}$ satisfies the boundary condition:

$$\hat{\mathbf{n}} \times \mathbf{E}^t(\mathbf{r}) = \mathbf{0}, \ \mathbf{r} \in S \tag{4}$$

where $\hat{\mathbf{n}}$ is the unit vector perpendicular to the outer surface S. In view of the Drude-Born-Fedorov constitutive relations [1], the total exterior electromagnetic field satisfies in the source-free region Ω^+ the modified Maxwell equations

$$\nabla \times \mathbf{E}^t_{\mathbf{r}_0}(\mathbf{r}) = \beta\gamma^2\mathbf{E}^t_{\mathbf{r}_0}(\mathbf{r}) + {} + \mathrm{i}\omega\mu(\frac{\gamma}{k})^2\mathbf{H}^t_{\mathbf{r}_0}(\mathbf{r}) \tag{5}$$

$$\nabla \times \mathbf{H}^t_{\mathbf{r}_0}(\mathbf{r}) = \beta\gamma^2\mathbf{H}^t_{\mathbf{r}_0}(\mathbf{r}) + {} - \mathrm{i}\omega\varepsilon(\frac{\gamma}{k})^2\mathbf{E}^t_{\mathbf{r}_0}(\mathbf{r}) \tag{6}$$

where

$$k^2 = \omega^2\varepsilon\mu, \quad \gamma^2 = \frac{k^2}{1 - k^2\beta^2} \tag{7}$$

with $|\beta k| < 1$, [1].

We note that (in contrast to the non-chiral case) k is not a wave number, but just a parameter without physical significance which has dimensions of inverse length and $\omega > 0$ is the angular frequency. The system of relations (5), (6) can be written as

$$\begin{bmatrix}\nabla\times\mathbf{E}^t_{\mathbf{r}_0} \\ \nabla\times\mathbf{H}^t_{\mathbf{r}_0}\end{bmatrix} = (\frac{\gamma}{k})^2\begin{bmatrix}k^2\beta & \mathrm{i}\omega\mu \\ -\mathrm{i}\omega\varepsilon & k^2\beta\end{bmatrix}\begin{bmatrix}\mathbf{E}^t_{\mathbf{r}_0} \\ \mathbf{H}^t_{\mathbf{r}_0}\end{bmatrix} \tag{8}$$

Beltrami Fields

As it is well known [1], [10], in chiral media LCP and RCP waves can both propagate independently and with different phase speeds. So, we consider the Bohren [9], decomposition of $\mathbf{E}_{\mathbf{r}_0}$ and $\mathbf{H}_{\mathbf{r}_0}$ into suitable Beltrami fields $\mathbf{Q}_{L,\mathbf{r}_0}$ and $\mathbf{Q}_{R,\mathbf{r}_0}$, as follows

$$\mathbf{E}^t_{\mathbf{r}_0} = \mathbf{Q}_{L,\mathbf{r}_0} + \mathbf{Q}_{R,\mathbf{r}_0} \tag{9}$$

$$\mathbf{H}^t_{\mathbf{r}_0} = \frac{1}{\mathrm{i}\eta}(\mathbf{Q}_{L,\mathbf{r}_0} - \mathbf{Q}_{R,\mathbf{r}_0}) \tag{10}$$

where $\eta = (\mu/\varepsilon)^{1/2}$ is the impedance of the material.

Because the transformation Bohren the relation (8) is written:

$$\begin{bmatrix}\nabla \times \mathbf{Q}_{L,\mathbf{r}_0} \\ \nabla \times \mathbf{Q}_{R,\mathbf{r}_0}\end{bmatrix} = \begin{bmatrix}\gamma_L & 0 \\ 0 & -\gamma_R\end{bmatrix}\begin{bmatrix}\mathbf{Q}_{L,\mathbf{r}_0} \\ \mathbf{Q}_{R,\mathbf{r}_0}\end{bmatrix} \tag{11}$$

or equivalent

$$\begin{cases}\nabla \times \mathbf{Q}_{L,\mathbf{r}_0} = \gamma_L\mathbf{Q}_{L,\mathbf{r}_0} \\ \nabla \times \mathbf{Q}_{R,\mathbf{r}_0} = -\gamma_R\mathbf{Q}_{R,\mathbf{r}_0}\end{cases} \tag{12}$$

where γ_L, γ_R are wave numbers for Beltrami fields $\mathbf{Q}_{L,\mathbf{r}_0}, \mathbf{Q}_{R,\mathbf{r}_0}$ and are given by,

$$\gamma_L = \frac{k}{1 - k\beta}, \gamma_R = \frac{k}{1 + k\beta} \tag{13}$$

respectively. Applying the transformation Bohren we split the field $\mathbf{E}^t_{\mathbf{r}_0}$, $\mathbf{H}^t_{\mathbf{r}_0}$ into the Beltrami fields $\mathbf{Q}_L$, $\mathbf{Q}_R$, so that the problem (2)-(6) becomes equivalent to the following problem: We are looking for $\mathbf{Q}_L$, $\mathbf{Q}_R$ in Ω^+ such that

$$\nabla\times\mathbf{Q}_L(\mathbf{r}) = \gamma_L\mathbf{Q}_L(\mathbf{r}),\ \ \mathbf{r}\in\Omega^+ \tag{14}$$

$$\nabla\times\mathbf{Q}_R(\mathbf{r}) = -\gamma_R\mathbf{Q}_R(\mathbf{r}),\ \ \mathbf{r}\in\Omega^+ \tag{15}$$

$$\hat{\mathbf{n}}\times\mathbf{Q}_L(\mathbf{r}) = \mathrm{i}\sqrt{\frac{\mu}{\varepsilon}}\,\hat{\mathbf{n}}\times\mathbf{Q}_R(\mathbf{r}),\ \ \mathbf{r}\in S \tag{16}$$

$$\hat{\mathbf{r}}\times\mathbf{Q}_L(\mathbf{r}) + \mathrm{i}\mathbf{Q}_L(\mathbf{r}) = o(\frac{1}{r}),\ r\to\infty \tag{17}$$

$$\hat{\mathbf{r}}\times\mathbf{Q}_R(\mathbf{r}) - \mathrm{i}\mathbf{Q}_R(\mathbf{r}) = o(\frac{1}{r}),\ r\to\infty \tag{18}$$

Spherical Beltrami Fields

So, we consider the Bohren decomposition of $\mathbf{E}_{\mathbf{r}_0}^{inc}$ and $\mathbf{H}_{\mathbf{r}_0}^{inc}$ into suitable incident spherical Beltrami fields $\mathbf{Q}_{L,\mathbf{r}_0}^{inc}$ and $\mathbf{Q}_{R,\mathbf{r}_0}^{inc}$ which have the form [6], [7],

$$\mathbf{Q}_{L,\mathbf{r}_0}^{inc}(\mathbf{r}\,|\,\hat{\mathbf{p}}_L) = A_L\tilde{\mathbf{B}}_L(\mathbf{r},\mathbf{r}_0)\cdot\hat{\mathbf{p}}_L \tag{19}$$

$$\mathbf{Q}_{R,\mathbf{r}_0}^{inc}(\mathbf{r}\,|\,\hat{\mathbf{p}}_R) = A_R\tilde{\mathbf{B}}_R(\mathbf{r},\mathbf{r}_0)\cdot\hat{\mathbf{p}}_R \tag{20}$$

where

$$\tilde{\mathbf{B}}_L(\mathbf{r},\mathbf{r}_0) = \frac{\mathrm{i}k\gamma_L}{8\pi\gamma^2}(\gamma_L\tilde{\mathbf{I}} + \frac{1}{\gamma_L}\nabla\nabla + \nabla\times\tilde{\mathbf{I}})h(\gamma_L\left|\mathbf{r}-\mathbf{r}_0\right|) \tag{21}$$

$$\tilde{\mathbf{B}}_R(\mathbf{r},\mathbf{r}_0) = \frac{\mathrm{i}k\gamma_R}{8\pi\gamma^2}(\gamma_R\tilde{\mathbf{I}} + \frac{1}{\gamma_R}\nabla\nabla - \nabla\times\tilde{\mathbf{I}})h(\gamma_R\left|\mathbf{r}-\mathbf{r}_0\right|) \tag{22}$$

and $h(x) = \mathrm{e}^{\mathrm{i}x}/(\mathrm{i}x)$ is the zeroth order spherical Hankel function of the first kind and $\tilde{\mathbf{I}} = \hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\boldsymbol{\psi}}\hat{\boldsymbol{\psi}} + \hat{\mathbf{z}}\hat{\mathbf{z}}$ is the identity dyadic. We recall that $\tilde{\mathbf{B}}_L$ and $\tilde{\mathbf{B}}_R$ are the fundamental Green dyadics, for the Beltrami fields [1], [11]. The constant unit vectors $\hat{\mathbf{p}}_L$ and $\hat{\mathbf{p}}_R$ are assumed to satisfy the relations:

$$\begin{aligned}&\hat{\mathbf{r}}_0\cdot\hat{\mathbf{p}}_L = \hat{\mathbf{r}}_0\cdot\hat{\mathbf{p}}_R = 0\\ &\hat{\mathbf{r}}_0\times\hat{\mathbf{p}}_L = \mathrm{i}\hat{\mathbf{p}}_L\\ &\hat{\mathbf{r}}_0\times\hat{\mathbf{p}}_R = -\,\mathrm{i}\hat{\mathbf{p}}_R\end{aligned} \tag{23}$$

The constants A_L and A_R are evaluated so that as the location of the point source goes to infinity along the ray in the direction $\hat{\mathbf{r}}_0$, the spherical fields degenerate into plane LCP and RCP Beltrami fields propagated in a direction from $\mathbf{r}_0$ towards O, with polarizations $\hat{\mathbf{p}}_L$ and $\hat{\mathbf{p}}_R$, respectively. Using the asymptotic forms:

$$\begin{aligned}&\left|\mathbf{r}-\mathbf{r}_0\right| = r_0 - \hat{\mathbf{r}}_0\cdot\mathbf{r} + O(\frac{1}{r_0}),\\ &\frac{\mathbf{r}-\mathbf{r}_0}{\left|\mathbf{r}-\mathbf{r}_0\right|} = -\,\hat{\mathbf{r}}_0 + O(\frac{1}{r_0}),\\ &\qquad r_0\to\infty\end{aligned} \tag{24}$$

in the relations (19) and (20) we obtain [6], [7],

$$\begin{aligned}\mathbf{Q}_{j,\mathbf{r}_0}^{inc}(\mathbf{r}\,|\,\hat{\mathbf{p}}_j) = A_j\frac{2e^{-\mathrm{i}\gamma_j r_0}}{r_0}\tilde{\mathbf{K}}_j(-\hat{\mathbf{r}}_0)\cdot&\\ \cdot\hat{\mathbf{p}}_j e^{-\mathrm{i}\gamma_j\hat{\mathbf{r}}_0\cdot\mathbf{r}} + O(\frac{1}{r_0^2}),&\\ r_0\to\infty&\end{aligned} \tag{25}$$

for $j = L, R$, where

$$\tilde{\mathbf{K}}_L(-\hat{\mathbf{r}}_0) = \frac{1}{2}(\tilde{\mathbf{I}} - \hat{\mathbf{r}}_0\hat{\mathbf{r}}_0 - \mathrm{i}\hat{\mathbf{r}}_0\times\tilde{\mathbf{I}}) \tag{26}$$

$$\tilde{\mathbf{K}}_R(-\hat{\mathbf{r}}_0) = \frac{1}{2}(\tilde{\mathbf{I}} - \hat{\mathbf{r}}_0\hat{\mathbf{r}}_0 + \mathrm{i}\hat{\mathbf{r}}_0\times\tilde{\mathbf{I}}) \tag{27}$$

In view of (23) the dyadics $\tilde{\mathbf{K}}_L$ and $\tilde{\mathbf{K}}_R$ satisfy the relations

$$\begin{cases}\hat{\mathbf{p}}_L = \tilde{\mathbf{K}}_L(-\hat{\mathbf{r}}_0)\cdot\hat{\mathbf{p}}_L\\ \hat{\mathbf{p}}_R = \tilde{\mathbf{K}}_R(-\hat{\mathbf{r}}_0)\cdot\hat{\mathbf{p}}_R\end{cases} \tag{28}$$

Hence, if we take

$$\begin{cases}A_L = r_0 e^{-\mathrm{i}\gamma_L r_0}\dfrac{4\pi\gamma^2}{k\gamma_L}\\ A_R = r_0 e^{-\mathrm{i}\gamma_R r_0}\dfrac{4\pi\gamma^2}{k\gamma_R}\end{cases} \tag{29}$$

then

$$\begin{aligned}\lim_{r_0\to\infty}\mathbf{Q}_{L,\mathbf{r}_0}^{inc}(\mathbf{r}\,|\,\hat{\mathbf{p}}_j) = e^{-\mathrm{i}\gamma_L\hat{\mathbf{r}}_0\cdot\mathbf{r}}\hat{\mathbf{p}}_j =&\\ = \mathbf{Q}_j^{inc}(\mathbf{r}; -\hat{\mathbf{r}}_0, \hat{\mathbf{p}}_j)&\end{aligned} \tag{30}$$

It is convenient to write the incident spherical Beltrami fields as [6], [7],

$$\mathbf{Q}_{L,\mathbf{r}_0}^{inc}(\mathbf{r}\,|\,\hat{\mathbf{p}}_L) = \tilde{\boldsymbol{\Gamma}}_L(\frac{h(\gamma_L u)}{h(\gamma_L r_0)})\cdot\hat{\mathbf{p}}_L \tag{31}$$

$$\mathbf{Q}_{R,\mathbf{r}_0}^{inc}(\mathbf{r}\,|\,\hat{\mathbf{p}}_R) = \tilde{\boldsymbol{\Gamma}}_R(\frac{h(\gamma_R u)}{h(\gamma_R r_0)})\cdot\hat{\mathbf{p}}_R \tag{32}$$

where

$$\tilde{\boldsymbol{\Gamma}}_L = \frac{1}{2\gamma_L}(\gamma_L\tilde{\mathbf{I}} + \frac{1}{\gamma_L}\nabla\nabla + \nabla\times\tilde{\mathbf{I}}) \tag{33}$$

$$\tilde{\boldsymbol{\Gamma}}_R = \frac{1}{2\gamma_R}(\gamma_R\tilde{\mathbf{I}} + \frac{1}{\gamma_R}\nabla\nabla - \nabla\times\tilde{\mathbf{I}}) \tag{34}$$

and $u = \left|\mathbf{r}-\mathbf{r}_0\right|$. Using asymptotic forms (24) for $r\to\infty$ we obtain [6], [7],

$$\begin{aligned}\mathbf{Q}_{j,\mathbf{r}_0}^{inc}(\mathbf{r}\,|\,\hat{\mathbf{p}}_j) = \mathbf{F}_{j,\mathbf{r}_0}^{inc}(\hat{\mathbf{r}}\,|\,\hat{\mathbf{p}}_j)h(\gamma_j r) +&\\ + O(\frac{1}{r^2})&\end{aligned} \tag{35}$$

where

$$\mathbf{F}_{j,\mathbf{r}_0}^{inc}(\hat{\mathbf{r}}\,|\,\hat{\mathbf{p}}_j) = \frac{e^{-\mathrm{i}\gamma_j\hat{\mathbf{r}}\cdot\mathbf{r}_0}}{h(\gamma_j r_0)}\tilde{\mathbf{K}}_l(\hat{\mathbf{r}})\cdot\hat{\mathbf{p}}_j \tag{36}$$

are the far-field patterns of the point source incident Beltrami fields, which satisfy the relations

$$\hat{\mathbf{r}}\cdot\mathbf{F}_{j,\mathbf{r}_0}^{inc}(\hat{\mathbf{r}}\,|\,\hat{\mathbf{p}}_j) = 0 \tag{37}$$

The scattered electric field $\mathbf{E}_{\mathbf{r}_0}^{sc}$ will be dependent on the polarization $\hat{\mathbf{p}}_L$, $\hat{\mathbf{p}}_R$ and will be have the decomposition

$$\begin{aligned}\mathbf{E}_{\mathbf{r}_0}^{sc}(\mathbf{r}\,|\,\hat{\mathbf{p}}_L,\hat{\mathbf{p}}_R) = \mathbf{Q}_{L,\mathbf{r}_0}^{sc}(\mathbf{r}\,|\,\hat{\mathbf{p}}_L,\hat{\mathbf{p}}_R)&\\ + \mathbf{Q}_{R,\mathbf{r}_0}^{sc}(\mathbf{r}\,|\,\hat{\mathbf{p}}_L,\hat{\mathbf{p}}_R)&\end{aligned} \tag{38}$$

where $\mathbf{Q}^{sc}_{L,\mathbf{r}_0}(\mathbf{r}\,|\,\hat{\mathbf{p}}_L,\hat{\mathbf{p}}_R)$ and $\mathbf{Q}^{sc}_{R,\mathbf{r}_0}(\mathbf{r}\,|\,\hat{\mathbf{p}}_L,\hat{\mathbf{p}}_R)$ are the corresponding scattered Beltrami fields, which have the following behavior, when $r\to\infty$ [1], [10], [12]

$$\mathbf{Q}^{sc}_{j,\mathbf{r}_0}(\mathbf{r}\,|\,\hat{\mathbf{p}}_L,\hat{\mathbf{p}}_R)=h(\gamma_j r)\mathbf{g}_{j,\mathbf{r}_0}(\hat{\mathbf{r}})+ \\ +O(\frac{1}{r^2}) \tag{39}$$

The functions $\mathbf{g}_{j,\mathbf{r}_0}(\hat{\mathbf{r}})$ given by

$$\mathbf{g}_{L,\mathbf{r}_0}(\hat{\mathbf{r}})=\frac{\mathrm{i}k\gamma_L}{8\pi\gamma^2}\tilde{\mathbf{K}}_L(\hat{\mathbf{r}})\cdot\int_S\hat{\mathbf{n}}\times(\gamma_L\nabla\times\mathbf{E}^{sc}_{\mathbf{r}_0}(\mathbf{r}')+\gamma^2\mathbf{E}^{sc}_{\mathbf{r}_0}(\mathbf{r}'))e^{-\mathrm{i}\gamma_L\hat{\mathbf{r}}\cdot\mathbf{r}'}ds(\mathbf{r}') \tag{40}$$

$$\mathbf{g}_{R,\mathbf{r}_0}(\hat{\mathbf{r}})=\frac{\mathrm{i}k\gamma_R}{8\pi\gamma^2}\tilde{\mathbf{K}}_R(\hat{\mathbf{r}})\cdot\int_S\hat{\mathbf{n}}\times(\gamma_R\nabla\times\mathbf{E}^{sc}_{\mathbf{r}_0}(\mathbf{r}')-\gamma^2\mathbf{E}^{sc}_{\mathbf{r}_0}(\mathbf{r}'))e^{-\mathrm{i}\gamma_R\hat{\mathbf{r}}\cdot\mathbf{r}'}ds(\mathbf{r}') \tag{41}$$

are the LCP and RCP far-field patterns, respectively [10], and they are dependent also on the polarization $\hat{\mathbf{p}}_L$ and $\hat{\mathbf{p}}_R$. We note that the far-field patterns $\mathbf{g}_{j,\mathbf{r}_0}(\hat{\mathbf{r}})$ satisfy the relations

$$\begin{cases}\hat{\mathbf{r}}\cdot\mathbf{g}_{j,\mathbf{r}_0}(\hat{\mathbf{r}})=0\\ \hat{\mathbf{r}}\times\mathbf{g}_{L,\mathbf{r}_0}(\hat{\mathbf{r}})=-\mathrm{i}\mathbf{g}_{L,\mathbf{r}_0}(\hat{\mathbf{r}})\\ \hat{\mathbf{r}}\times\mathbf{g}_{R,\mathbf{r}_0}(\hat{\mathbf{r}})=\mathrm{i}\mathbf{g}_{R,\mathbf{r}_0}(\hat{\mathbf{r}})\end{cases} \tag{42}$$

Reciprocity and General Scattering Theorems

For two vector functions **u** and **v** we introduce the bilinear form [10],

$$\{\mathbf{u},\mathbf{v}\}_S= \\ =\int_S\hat{\mathbf{n}}\cdot(\mathbf{u}\times\nabla\times\mathbf{v}-\mathbf{v}\times\nabla\times\mathbf{u})dS- \\ -2\beta\gamma^2\int_S\hat{\mathbf{n}}\cdot(\mathbf{u}\times\mathbf{v})dS \tag{43}$$

where S is the surface of the scatterer Ω^- and $\hat{\mathbf{n}}$ is the outward unit vector on S. We, also, consider two locations for the point source, **a** and **b** from which the time-harmonic incident spherical electric waves

$$\mathbf{E}^{inc}_{\boldsymbol{\sigma}}(\mathbf{r}\,|\,\hat{\mathbf{p}}_L,\hat{\mathbf{p}}_R)= \\ =\mathbf{Q}^{inc}_{L,\boldsymbol{\sigma}}(\mathbf{r}\,|\,\hat{\mathbf{p}}_L)+\mathbf{Q}^{inc}_{R,\boldsymbol{\sigma}}(\mathbf{r}\,|\,\hat{\mathbf{p}}_R) \tag{44}$$

for $\boldsymbol{\sigma}=\mathbf{a},\mathbf{b}$, emanate. $\mathbf{E}^{sc}_{\boldsymbol{\sigma}}$ and the corresponding scattered electric waves $\mathbf{E}^{sc}_{\boldsymbol{\sigma}}$ of form (38), have the Bohren decomposition of (9) in terms of LCP and RCP Beltrami fields. In particular, we have

$$\begin{cases}\mathbf{E}^{inc}_{\boldsymbol{\sigma}}=\mathbf{Q}^{inc}_{L,\boldsymbol{\sigma}}+\mathbf{Q}^{inc}_{R,\boldsymbol{\sigma}}\\ \mathbf{E}^{sc}_{\boldsymbol{\sigma}}=\mathbf{Q}^{sc}_{L,\boldsymbol{\sigma}}+\mathbf{Q}^{sc}_{R,\boldsymbol{\sigma}}\end{cases} \tag{45}$$

and the following properties for the fields $\mathbf{Q}^{inc}_{j,\boldsymbol{\sigma}}$ and $\mathbf{Q}^{sc}_{j,\boldsymbol{\sigma}}$ [6], [7]:

$$\{\overline{\mathbf{Q}}^{inc}_{L,\mathbf{a}}(\mathbf{r};\hat{\mathbf{p}}_L),\mathbf{Q}^{sc}_{L,\mathbf{b}}(\mathbf{r}\,|\,\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R)\}_S=\frac{2\gamma^2}{k}\int_S\hat{\mathbf{n}}\cdot(\overline{\mathbf{Q}}^{inc}_{L,\mathbf{a}}(\mathbf{r};\hat{\mathbf{p}}_L)\times\mathbf{Q}^{sc}_{L,\mathbf{b}}(\mathbf{r}\,|\,\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R))ds(\mathbf{r}) \tag{46}$$

$$\{\overline{\mathbf{Q}}^{inc}_{L,\mathbf{a}}(\mathbf{r};\hat{\mathbf{p}}_L),\mathbf{Q}^{sc}_{R,\mathbf{b}}(\mathbf{r}\,|\,\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R)\}_S=\mathbf{0} \tag{47}$$

$$\{\overline{\mathbf{Q}}^{inc}_{R,\mathbf{a}}(\mathbf{r};\hat{\mathbf{p}}_R),\mathbf{Q}^{sc}_{R,\mathbf{b}}(\mathbf{r}\,|\,\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R)\}_S=-\frac{2\gamma^2}{k}\int_S\hat{\mathbf{n}}\cdot(\overline{\mathbf{Q}}^{inc}_{R,\mathbf{a}}(\mathbf{r};\hat{\mathbf{p}}_R)\times\mathbf{Q}^{sc}_{R,\mathbf{b}}(\mathbf{r}\,|\,\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R))ds(\mathbf{r}) \tag{48}$$

$$\{\overline{\mathbf{Q}}^{inc}_{R,\mathbf{a}}(\mathbf{r};\hat{\mathbf{p}}_R),\mathbf{Q}^{sc}_{L,\mathbf{b}}(\mathbf{r}\,|\,\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R)\}_S=\mathbf{0} \tag{49}$$

Now a "reciprocity theorem" for spherical electromagnetic waves in chiral media is formulated as follows [6], [7]:

For any two incident spherical electric waves $\mathbf{E}^{inc}_{\mathbf{a}}(\mathbf{r}\,|\,\hat{\mathbf{p}}_L,\hat{\mathbf{p}}_R)$ and $\mathbf{E}^{inc}_{\mathbf{b}}(\mathbf{r}\,|\,\hat{\mathbf{p}}_L,\hat{\mathbf{p}}_R)$ of (44) and for any scatterer in a homogeneous isotropic chiral medium, we have:

$$\frac{ae^{-\mathrm{i}\gamma_L a}}{\gamma_L}\mathbf{Q}^{sc}_{L,\mathbf{b}}(\mathbf{a}\,|\,\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R)\cdot\hat{\mathbf{p}}_L+ \\ +\frac{ae^{-\mathrm{i}\gamma_R a}}{\gamma_R}\mathbf{Q}^{sc}_{R,\mathbf{b}}(\mathbf{a}\,|\,\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R)\cdot\hat{\mathbf{p}}_R= \\ =\frac{be^{-\mathrm{i}\gamma_L b}}{\gamma_L}\mathbf{Q}^{sc}_{L,\mathbf{a}}(\mathbf{b}\,|\,\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R)\cdot\hat{\mathbf{q}}_L+ \\ +\frac{be^{-\mathrm{i}\gamma_R b}}{\gamma_R}\mathbf{Q}^{sc}_{R,\mathbf{a}}(\mathbf{b}\,|\,\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R)\cdot\hat{\mathbf{q}}_R \tag{50}$$

Let $\mathbf{E}^{inc}_{\mathbf{a}}(\mathbf{r}\,|\,\hat{\mathbf{p}}_L,\hat{\mathbf{p}}_R)$ and $\mathbf{E}^{inc}_{\mathbf{b}}(\mathbf{r}\,|\,\hat{\mathbf{p}}_L,\hat{\mathbf{p}}_R)$ be two incident spherical waves of the form (44). We define spherical far-field pattern generators for LCP and RCP spherical Beltrami fields by

$$\mathbf{G}_{j,\mathbf{b}}(\mathbf{a}\,|\,\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R)=\frac{ae^{\mathrm{i}\gamma_L a}}{\mathrm{i}\gamma_j}\cdot \\ \cdot\{\frac{1}{4\pi}\int_{S^2}\mathbf{g}_{j,\mathbf{b}}(\mathbf{a}|\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R)\cdot\overline{\tilde{\mathbf{K}}_j(\hat{\mathbf{r}})}e^{\mathrm{i}\gamma_A\hat{\mathbf{r}}\cdot\mathbf{a}}ds(\hat{\mathbf{r}})- \\ -\mathbf{Q}^{sc}_{j,\mathbf{b}}(\mathbf{a}\,|\,\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R)\} \tag{51}$$

for $j=L,R$.

This terminology and definition is appropriate because when both the observation point and the source goes to infinity, $\mathbf{G}_{j,\mathbf{b}}(\mathbf{a}\,|\,\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R)$, reduce to the far-field patterns for an incident plane electric wave propagating in the direction $-\hat{\mathbf{b}}$ and of polarizations $\hat{\mathbf{q}}_L$ for LCP and $\hat{\mathbf{q}}_R$ for RCP fields. Far-field pattern generators in acoustic and achiral electromagnetic scattering have been defined in [13]. When the point sources are transferred at infinity, the generators $\mathbf{G}_{j,\mathbf{b}}(\mathbf{a}\,|\,\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R)$ are transformed into far-field patterns scattering spherical electrical wave, namely [6], [7],

$$\lim_{a\to\infty} \mathbf{G}_{j,\mathbf{b}}(\mathbf{a}|\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R)=\boldsymbol{g}_{j,\mathbf{b}}(-\hat{\mathbf{a}}|\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R) \quad (52)$$

Using this notation, the "general scattering theorem" for spherical electric waves in chiral media is formulated as follows [6], [7]:

Let $\mathbf{E}_{\mathbf{a}}^{inc}(\mathbf{r}\,|\,\hat{\mathbf{p}}_L,\hat{\mathbf{p}}_R)$ and $\mathbf{E}_{\mathbf{b}}^{inc}(\mathbf{r}\,|\,\hat{\mathbf{p}}_L,\hat{\mathbf{p}}_R)$ be two spherical electric waves of (44) incident upon a scatterer in a chiral medium. Then it is valid

$$\begin{aligned}&\mathbf{G}_{L,\mathbf{b}}(\mathbf{a}\,|\,\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R)\cdot\hat{\mathbf{p}}_L+\mathbf{G}_{R,\mathbf{b}}(\mathbf{a}\,|\,\hat{\mathbf{q}}_L,\hat{\mathbf{q}}_R)\cdot\hat{\mathbf{p}}_R+\\&+\overline{\mathbf{G}_{L,\mathbf{a}}(\mathbf{b}\,|\,\hat{\mathbf{p}}_L,\hat{\mathbf{p}}_R)}\cdot\hat{\mathbf{q}}_L+\overline{\mathbf{G}_{R,\mathbf{a}}(\mathbf{b}\,|\,\hat{\mathbf{p}}_L,\hat{\mathbf{p}}_R)}\cdot\hat{\mathbf{q}}_R=\\&=-\frac{1}{2\pi}\{\int_{S^2}\frac{1}{\gamma_L^2}\overline{\mathbf{F}_{L,\mathbf{a}}^{sc}(\hat{\mathbf{r}})}\cdot\mathbf{F}_{L,\mathbf{b}}^{sc}(\hat{\mathbf{r}})\,dS(\hat{\mathbf{r}})+\\&+\int_{S^2}\frac{1}{\gamma_R^2}\overline{\mathbf{F}_{R,\mathbf{a}}^{sc}(\hat{\mathbf{r}})}\cdot\mathbf{F}_{R,\mathbf{b}}^{sc}(\hat{\mathbf{r}})\,dS(\hat{\mathbf{r}})\} \qquad (53)\end{aligned}$$

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Self-Standing End-Fed Geometrically Uniform Linear Arrays: Analysis, Design, Construction, Measurements and FLOSS

K.Th. Kondylis, N.I. Yannopoulou, P.E. Zimourtopoulos *

Antennas Research Group, Athens, Greece [1]
Antennas Research Group, Austria — Hellas [2, 3]

Abstract

The array factor of a both geometrically and electrically uniform array is the simple formula for the complex geometric progression sum. This fact, although results in the simplest of all possible analytical designs, obviously does not in the least simplify the complicated practical problem of feeding the array elements using multiple driving points. In order to begin the examination of uniform linear arrays with a single driving point, this paper presents a compact study of the end-fed space arrays with application to geometrically uniform, self-standing linear arrays of parallel dipoles. A number of test array models were simulated, constructed and their radiation pattern was then measured. The experimental and computational results were found to be in good agreement. The developed software applications are available through the Internet as FLOSS Free Libre Open Source Software.

Keywords

End-fed, single driving-point, self-standing arrays

Introduction

A space array of $1 \leq k \leq N$ parallel, arbitrarily shaped, dipoles with identical current density distributions, has a complex vector radiation pattern $E = AG$ (by applying the "Principle of Radiation Patterns Multiplication", i.e. by the linear property of the volume integral), where G is the radiation pattern of the first dipole (which "generates the array"; the "Generator Pattern"), and A is the Array Factor,

$$A = \sum_{k=1}^{N} \left(\frac{I_k}{I_1}\right) e^{i\beta\left(R_{k_r} - R_{1_r}\right)} \qquad (1)$$

i.e. the complex numerical radiation pattern of N invented "isotropic point sources", each of current I_k and pointed by the dipole center vector R_k, with projection R_{k_r} to the unit direction vector r. Linear Arrays have dipole centers on a straight line. Fully Uniform Linear Arrays are both geometrically uniform, i.e. the dipoles are equidistant, and electrically uniform, i.e. the consecutive dipole currents are of equal amplitude and constant phase difference [1].

End-Fed Arrays

Perhaps, the simplest practical array is the one constructed from a linear two-wire transmission line that supports the arms of parallel, linear, symmetrical dipoles, vertical to its plane. In order to operate the line as a balanced one, it is end-fed through a coaxial line balun from a coaxial connector, which at the same time supports the weight of the whole, self-standing, array. A possible analysis of such an array is shown in Fig. 1.

In this one port linear network, the number of circuit voltage and current variables is equal to the sum of 2 variables for the input port plus 2N variables for the dipole ports plus 4(N−1) variables for the line segment ports, that is a total number of 6N−2 variables. The number of the linear relations between these variables is equal to the sum of 3 relations, for the generator dipole

$$1 \leq N \ : \ V = V_1 = {}_iV_1, \quad I = I_1 + {}_iI_1 \qquad (2)$$

plus 2 relations, for the last dipole

$$2 \leq N \ : \ {}_oV_{N-1} = V_N, \quad {}_oI_{N-1} + I_N = 0 \qquad (3)$$

plus 3(N−2) relations, for the intermediate dipoles

$$3 \leq N, \ 1 \leq k \leq N-2 \ : \ {}_oV_k = V_{k+1} = {}_iV_{k+1}, \quad {}_oI_k + I_{k+1} + {}_iI_{k+1} = 0 \qquad (4)$$

plus N relations, for the coupling between these dipoles

$$1 \leq k \leq N \ : \ V_k = \sum_{\mu=1}^{N} z_{k\mu} I_\mu \qquad (5)$$

plus 2(N−1) relations, for each one of the lossless transmission line segments of electrical length βl_k and characteristic impedance Z_{0k}

$$0 < \beta l_k \neq \nu\pi, \ \nu = 1,2,3,\ldots \ : \ \begin{bmatrix} {}_{ii}z_k & {}_{io}z_k \\ {}_{oi}z_k & {}_{oo}z_k \end{bmatrix} = -i\frac{Z_{0k}}{\sin\beta l_k}\begin{bmatrix} \cos\beta l_k & 1 \\ 1 & \cos\beta l_k \end{bmatrix} \qquad (6)$$

$$\beta l_k = \nu\pi, \ \nu = 2\mu + 1, \ \mu = 0,1,2,\ldots \ : \ {}_oV_k = -\,{}_iV_k, \quad {}_oI_k - {}_iI_k = 0 \qquad (7)$$

$$\beta l_k = \nu\pi,\ \nu = 2\mu + 2,\ \mu = 0,1,2,\ldots :\ {}_oV_k = {}_iV_k,\ \ {}_oI_k + {}_iI_k = 0 \qquad (8)$$

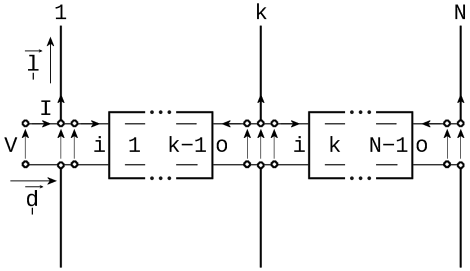


Fig. 1: End-fed linear array of linear dipoles

that is a total of 6N−3 linear relations to solve for 6N−2 variables, in terms of the source voltage V, which is considered as a parameter. Anyhow, the resulting current ratios are clearly independent of V.

Single Driving-Point Self-Standing Linear Array

The authors' group has limited available technical resources for antenna construction. This fact restricts the experimentation to thin-wire self-standing array models of a low total weight that is of a small total transmission ine length and of a small number of dipoles. Thus, the practical application of the analysis was carried-out for N = 2, 3 and 4 dipoles only. In order to demonstrate the procedure in use, the smallest case of N = 2 dipoles is presented in some detail. In Fig. 2, the resulting 6N−3 = 9 linear relations between 6N−2 = 9 variables + 1 parameter, are shown in a compact form, for the case of $\beta l_1 \neq \nu\pi, \nu = 1, 2, \ldots$ in (6), with each cell value to be the coefficient of the variable in the first row of its column in an implied summation, while, if $\beta l_1 = \nu\pi,\ \nu = 1, 2, \ldots$ then the two last rows with the gray background have to substituted by the rows of Fig. 3 or 4, according to the odd or even value of ν given in (7)-(8).

V_1	V_2	I_1	I_2	${}_iV_1$	${}_oV_1$	${}_iI_1$	${}_oI_1$	I	=	V
1									=	1
1				−1					=	0
		1				1		−1	=	0
	1				−1				=	0
			1				1		=	0
−1		Z_{11}	Z_{12}						=	0
	−1	Z_{21}	Z_{22}						=	0
				−1		${}_{ii}Z_1$	${}_{io}Z_1$		=	0
					−1	${}_{oi}Z_1$	${}_{oo}Z_1$		=	0

Fig. 2: The linear system of 9 relations for arrays with N = 2 and $\beta l_1 \neq \nu\pi, \nu = 1, 2, \ldots$

Obviously, the complexity of the expressions increase with the number of dipoles, from the simple, of 2 dipoles

$$I_{21} = \frac{I_2}{I_1} = \frac{Z_{12}\,{}_{oo}Z_1 - {}_{io}Z_1 Z_{22}}{{}_{io}Z_1^2 + {}_{io}Z_1 Z_{12} - {}_{oo}Z_1\left(Z_{22} + {}_{oo}Z_1\right)} \qquad (9)$$

in which the equality of self and mutual impedances have been taken into account, resulting from the system of Fig. 2, or of the simplest $I_2/I_1 = +1$ or $I_2/I_1 = -1$ of Fig. 3 or Fig. 4 respectively, to the most complex one for 4 dipoles, shown in Fig. 5, which covered about one and a half A4 page.

				1	1				=	0
						−1	1		=	0

Fig. 3: Last rows replacement for ν = 2μ + 1, μ = 0, 1, 2, ...

				−1	1				=	0
						1	1		=	0

Fig. 4: Last rows replacement for ν = 2μ + 2, μ = 0, 1, 2, ...

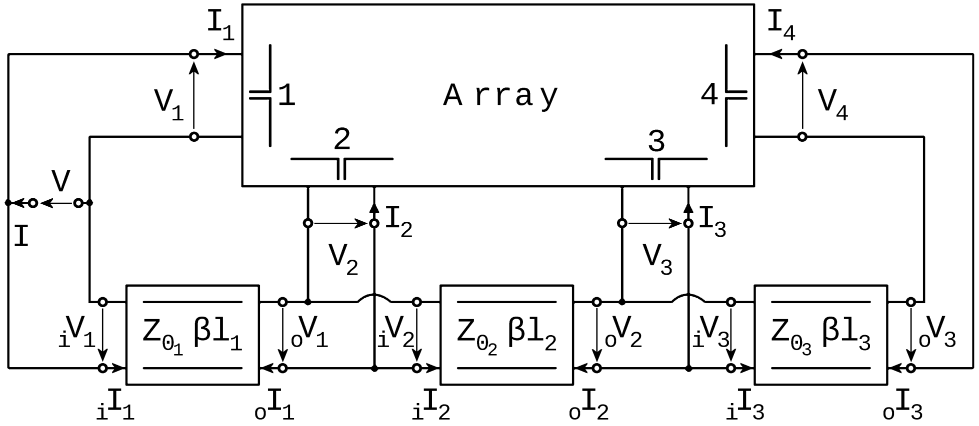


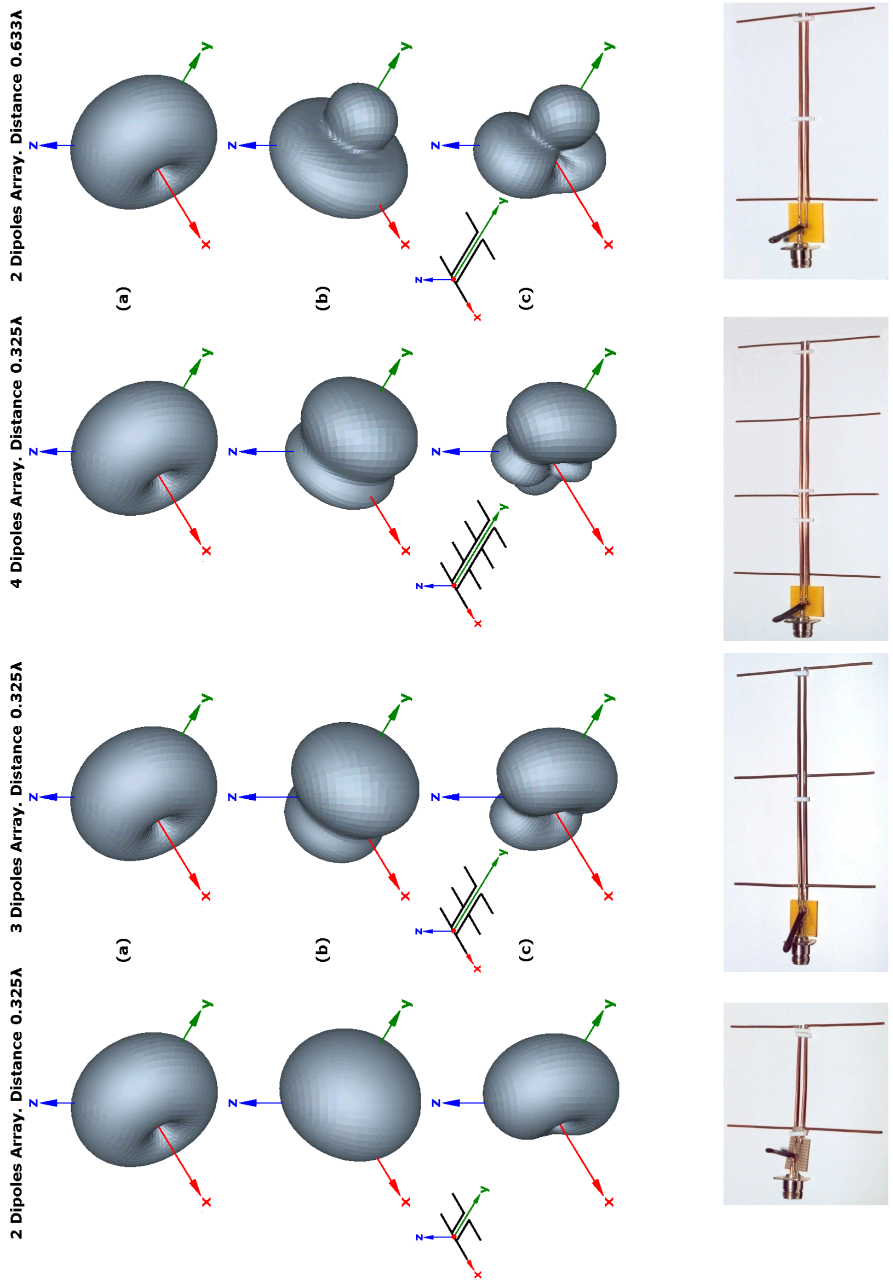
Fig. 5: Equivalent circuit of the 4 dipoles linear array

Three Visual Fortran applications were developed for the computation of current ratios. The GUI application form for N = 4 dipoles, is shown in Fig. 6 [2], [3]. In this form, the input data are the N−1 distances between dipoles, the dipole radius and length, the length, the characteristic impedance and the velocity factor of each transmission line segment. In addition to current ratios, each application produces the text files needed by the [RadPat4W] application of a FLOSS mini-Suite of tools, which plots in Virtual Reality the Generator, Array Factor, and Array 3D radiation pattern, as well as, their 2D main-plane cuts [4]. The deduced formulas for the determination of the 1 + 2 + 3 = 6 current ratios (I_k/I_1), were mechanically verified using Mathematica.

Array Design, Construction and Measurements

A number of dipole array designs were carried out using the developed applications and the [RichWire] simulation program, which is a fully analyzed, corrected and redeveloped edition of the original Moment Method thin-wire computer program [4], [5]. Eleven arrays were finally constructed and tested. The results for the current ratios of four selected arrays are shown in Tab. 1. The arrays were designed for operation at the frequency of 1.111 GHz. A two-wire transmission line, of $Z_0 = 200\ \Omega$ and velocity factor $vf = 1$ was constructed to feed the dipoles [6]. This balanced line was then connected to an unbalanced 50 Ω type-N/F base connector through a 4:1 balun made from a segment of RG-174U coaxial cable ($Z_0 = 50\ \Omega$, $vf = 0.66$) with total length λ/2.

The arrays were constructed by bare copper wire of 1 mm (0.0037λ) radius and they are self-standing using an orthogonal piece of a two-sided printed board (3 cm x 4.48 cm), on which the two-conductor line was soldered. A few Teflon spacers of low relative dielectric constant (≈ 2) were fabricated to fix the distance between the two line wires at 5.5 mm. The measurement system consists of a 50 Ω Vector Network Analyzer external to an anechoic chamber [7]. Each array under test was azimuthally rotated around its three main axes, by a 360° built positioner, under the developed software control of a built hardware controller. The stationary antenna was a UHF standard gain antenna [8]. Fig. 7 and Fig. 8 show the results for the 4 test arrays, in 3 groups of rows, as follows: 1st group: The screen captures of the produced Virtual Reality radiation patterns in dB for (a) Generator, (b) Array Factor, and (c) Dipole Array - 2nd group: The constructed model, and 3rd group: The 2D radiation pattern cuts by (d) xOy, (e) yOz, and (f) zOx main-planes. In Fig. 7, the measurements for the array of N = 2, dipoles with equidistance d = 0.325λ, were carried out with and without balun. In Fig. 8, the array of N = 2 dipoles with equidistance d = 0.633λ, has been designed to exhibit the maximum radiation pattern direction off the 3 main-axes.



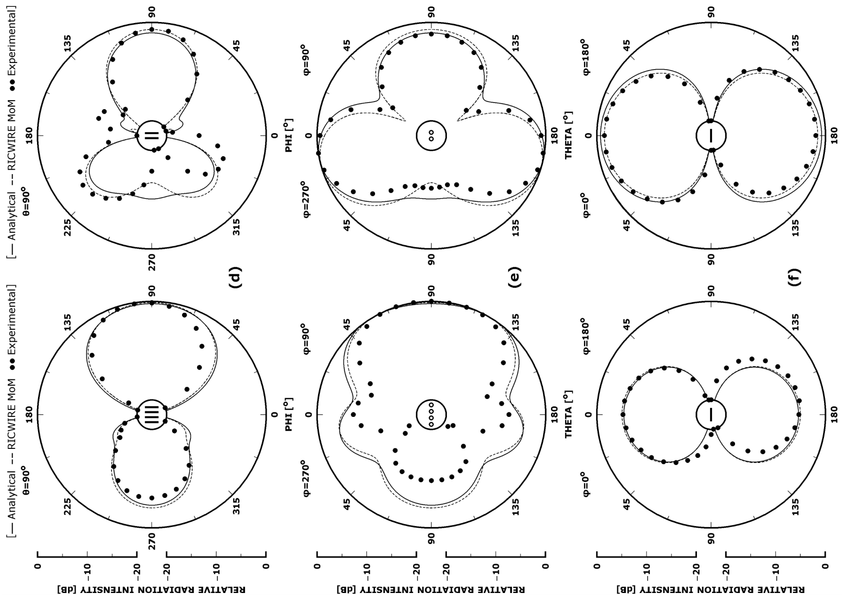


Fig. 8: Analysis, simulation and measurements for experimental arrays: N = 4 d = 0.325λ, N = 2 and d = 0.633λ

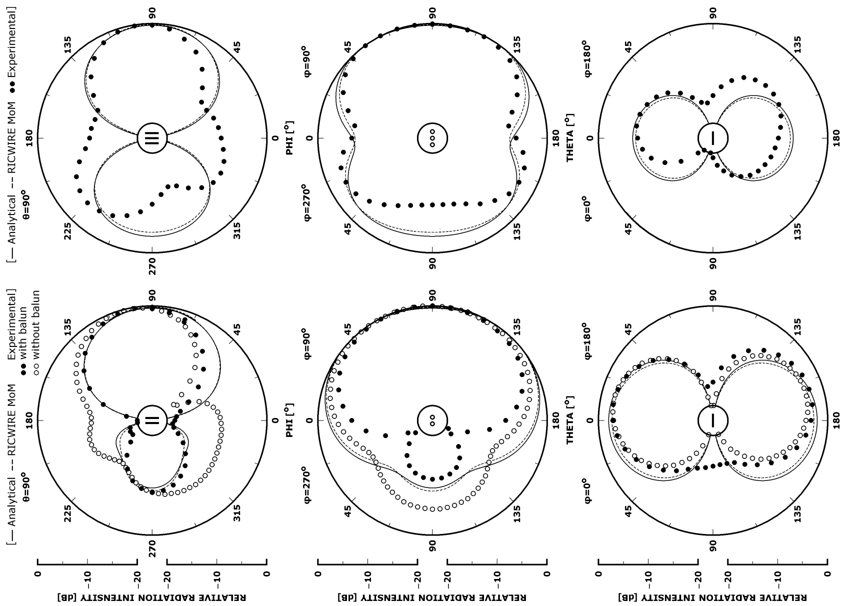
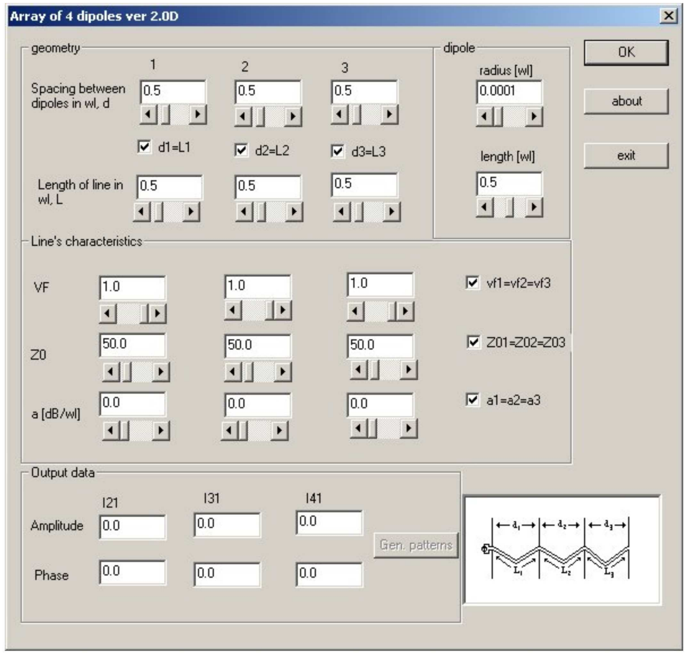


Fig. 7: Analysis, simulation and measurements for experimental arrays: N = 2, N = 3 and d = 0.325λ



geometry

	1	2	3
Spacing between dipoles in wl, d	0.5	0.5	0.5
	☑ d1=L1	☑ d2=L2	☑ d3=L3
Length of line in wl, L	0.5	0.5	0.5

dipole

radius [wl] 0.0001

length [wl] 0.5

Line's characteristics

VF	1.0	1.0	1.0	☑ vf1=vf2=vf3
Z0	50.0	50.0	50.0	☑ Z01=Z02=Z03
a [dB/wl]	0.0	0.0	0.0	☑ a1=a2=a3

Output data

	I21	I31	I41
Amplitude	0.0	0.0	0.0
Phase	0.0	0.0	0.0

Fig. 6: GUI for the analysis of a linear array of N = 4 dipoles

Tab. 1: The experimental array characteristics and current ratios

Dipoles	2	3		4			2
L/λ	0.5185	0.5		0.5			0.44
d/λ	0.325	0.325		0.325			0.633
Ratios	I_2/I_1	I_2/I_1	I_3/I_1	I_2/I_1	I_3/I_1	I_4/I_1	I_2/I_1
$\|I_k/I_1\|$	0.534	0.205	0.286	0.280	0.115	0.226	0.527
$\measuredangle I_k/I_1[°]$	–80.5	–114.9	173	–115.9	150.4	68.1	49.5

Conclusion

Although, the unavoidable mechanical supporting elements which exist in the anechoic chamber in the neighborhood of the antenna under test may affect the radiation pattern measurements, the observed differences in the 4 of the 12 patterns between the analysis and simulation results, on the one hand, and the measurements, on the other, have to be charged respectively: (1) On a cone-cut of the array radiation pattern, instead of the expected yOz main-plane cut, in Fig. 7(e), N = 2 and Fig. 8(e), N = 4, (2) On an inclination of the rotation axis relative to the expected linear polarization measurement plane, in Fig. 8(d), N = 2, and (3) On a loosed connection during the array rotation, in Fig. 7(d), N = 3. These conclusions are amplified by the careful study of the corresponding Virtual Reality space radiation patterns. Therefore, under the given measurement circumstances, the experimental and computational results were found to be in good agreement and no attempt was made to modify any design or repeat any measurement.

The results for the single driving-point self-standing fully Uniform Linear Arrays, i.e. those including the electrical uniformity, along with their application to the constrained pattern design will be presented in a future paper (Part 2).

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"A Study of End-Fed Arrays with Application to Single Driving-Point Self-Standing Linear Arrays"

Konstantinos Kondylis, Nikolitsa Yannopoulou, Petros Zimourtopoulos
"http://arxiv.org/abs/1002.3871"

Proceedings of 15th Conference on Microwave Techniques, COMITE 2010, Brno, Czech Republic, pp. 43-46

Follow-Up Research Paper

Not until now

Previous Publication in FUNKTECHNIKPLUS # JOURNAL

"The Very First Ever Made 3D/4D Virtual Laboratory for Antennas", Issue 3, pp. 7-18

* About The Authors

Konstantinos Kondylis, was born in Athens, Greece in 1976. He graduated from Electrical Engineering and Computer Engineering in 2000 and received a MSc in Antennas in 2002, both from Democritus University of Thrace, Xanthi, Greece. Member of Antennas Research Group since 2000, he worked as R&D Engineer for Siemens and Nokia Solutions and Networks. He is currently working as Sr. Project Engineer in Doha, Qatar.

Nikolitsa Yannopoulou, Issue 1, p. 15

Petros Zimourtopoulos, Issue 1, p. 15

Parameters Affecting the Lifetime of Transformer Oil in Distribution Transformers: Parameter Monitoring of 50 Transformers from the Athens Area

M.G. Danikas, E. Rapti, I. Liapis, A.B.B.Abd. Ghani *

Department of Electrical and Computer Engineering,
Democritus University of Thrace, Xanthi, Greece [1, 2]
Public Electricity Corporation, Athens, Greece [3]
High Voltage Cable Diagnostics, Distribution Unit TNB
Research Sdn. Bhd. Selangor, Malaysia [4]

Abstract

The aim of this paper is the study of various parameters affecting the ageing of transformer oil in distribution transformers of 20/0.4 kV. Fifty (50) samples of oil were taken from such transformers. The transformers function in the major Athens area, Greece. Parameters, such as breakdown strength, oil color, humidity, interfacial tension and tanδ were taken into account. Transformer ageing and lifetime are strictly related to the rate of ageing of the whole insulating system, and mainly of the oil.

Absence of transpiring system with silica gel has as a result the increase of oil humidity. The lengthy use of transformers under heavy load, and consequently under high temperatures, is a main factor for oil ageing and oxidation. Especially in the last few years, the increased loads required by the transformers which have to do also with the climatic changes, have as a result the additional stressing of the oil. Various arcs, resulting from short circuits in the network, have as a consequence the production of gases and sludge. Such gases and sludge influence in a negative way the insulating properties of the oil as well as its rate of ageing. The role of silica gel is stressed.

Keywords

Transformer oil, distribution transformer, diagnostic methods, breakdown voltage, dielectric strength

Introduction

Transformer oil is a most important component in transformers [1]. Transformer oil ageing has been studied as well as the parameters and factors influencing its behavior under a variety of stresses (electrical, thermal, etc.) [2]-[5]. Faults in power and distribution transformers are rare (of the order of 1% - 2% per year), but when they occur, they have very serious technical and economic consequences. They may even lead to dangerous situations for human life and the environment. Main factors affecting the acceleration of ageing of the insulation of a transformer are humidity, high temperature, oxidation and the acidity of its oil. The role of chemical byproducts of the transformer insulation is also very important for its ageing. There is no single measurement able to give a whole picture of the state of a transformer. It must be noted that the variety of the diagnostic methods used, does not have as an aim the prediction of the useful remaining lifetime of the insulation, but it tries to reveal the increasing probability of faults and the corresponding decrease of the insulation reliability. Needless to say, that reduced reliability implies also reduced remaining insulation lifetime.

With this in mind, in the context of the present paper, diagnostic methods were used in order to study the state of distribution transformers of the major Athens area. The transformers investigated were 20/0.4 kV. The whole work was carried with the aid of Public Power Company (PPC) of Greece, and more specifically with the aid of PPC Transformer Division. Oil was taken from 50 distribution transformers.

Diagnostic Methods

Warning signs about the state of a transformer are, among others, a big increase of partial discharges (>> 2500 pC), a visible deterioration because of foreign metallic and carbon particles, the presence of humidity in the solid insulation about 3-4 % and the presence of sludge.

Several diagnostic methods were used in order to see the quality of the transformers in question. The characterization of the oil color (DIN 51517 – ASTM D155) was performed through a device (chromometer) including standard glass disks and two glass jars with lid. The control of dielectric strength was measured by a typical Foster test cell, according to IEC 156/95 (Fig. 1).

The control of humidity in the oil was measured by a Metrohm – 684 KF Coulometer, which consisted of a glass container with a stirrer titration in which the reagent from container storage is added. The device is fully automated and once the experimenter gives the settings, it measures the moisture content of the oil. The measurements were performed according to IEC 814. The control of interfacial tension (ASTM D971 – 91) was performed via a tensimeter, which gives the value in dynes per centimeter in a direct reading (Fig. 2).

Fig. 1: Device for the measurement of dielectric strength

The device that performed measurements of tanδ and of resistivity, is the BAUR-DTL fully automated device for measuring dielectric losses of oils. Such a system has a fully automated process for measuring dielectric loss, relative dielectric constant and resistivity. The measurements were performed with a system counting tanδ values with maximum accuracy from 0.00001 to 4.0, measured according to IEC 247.

It is true that no single diagnostic method can give full information as to the state of a transformer. The aforementioned methods may give a better picture of its state.

Fig. 2: Tensimeter Cenco du Nouy

Experimental Results

The sampling was performed with due care, and according to general standard practice, i.e. during sampling there must not be any dust and humidity in the nearby space, the sampling cells must be clean and dry and they must be washed with oil from the transformer which is to be checked, the samples must be protected from light and they must be taken while the oil is hot but due time should pass before the various particles settle down. Sampling was done from the bottom of the transformers. The glass cells used for oil sampling were big enough so that they contained enough oil volume for additional measurements, if needed. No measurement was performed based on only one value or sample. All results must be verified with additional samples and measurements. Color measurements of the oil samples were based on specification DIN 51517 – ASTM D155 (Fig. 3). Dielectric strength measurements were based on specification IEC 156/95. Humidity measurements were based on specification IEC 814. Interfacial tension measurements were based on specification ASTM D971-91. Loss factor measurements were based on specification IEC 247.

Transformer oil is characterized as good, if its color is not dark, its breakdown voltage more than 40 kV, its humidity less than 10 ppm, its resistivity more than 3 GΩ·m, its loss factor 0.1, its interfacial tension more than 28 mN/m. It is acceptable if its breakdown voltage is between 30 to 40 kV, its humidity between 10 and 25 ppm, its resistivity between 0.2 and 3 GΩ·m, its loss factor between 0.1 and 0.5, its interfacial tension between 22 and 28 mN/m. In the case of an acceptable oil, samplings should be carried out more frequently, and the content in foreign particles and contained water should be controlled. If the breakdown voltage and humidity are near the limit values, then the oil should be filtered and cleaned. An oil is poor when its color is dark (the dark color is an indication of pollution or ageing), its breakdown voltage less than 30 kV (in this case the oil should be replaced or thoroughly cleaned), its humidity more than 25 ppm (in such a case the oil should be replaced or thoroughly cleaned), its resistivity less than 0.2 GΩ·m, its loss factor more than 0.5, its interfacial tension less than 22 mN/m (in this case a check should be carried out for the presence of sludge).

Generally it is suggested that in case one of the parameters mentioned (color, dielectric strength, humidity, interfacial tension, loss factor, resistivity) is acceptable, a more frequent sampling should be done in order to better monitor the oil quality. In the case any of the mentioned parameters is poor, cleaning or replacement of the oil is suggested. In any case, it is good to control all parameters concerned in order to have a clear picture of the state of the oil.

From the 50 oil samples investigated (taken from 50 distribution transformers), 3 of them showed breakdown voltage lower than 30 kV (i.e. percentage of 6%), 45 samples indicated humidity higher than 25 ppm (i.e. percentage 90%), 28 samples presented interfacial tension lower than 15 dynes/cm (percentage 56%), whereas 44 samples showed a color higher than 1 1/2 (percentage 88%) (Fig. 4). It was evident from the data – mentioned in detail in [6] – that the more the number of years in service, the more the number of samples with lower dielectric strength. This is due to the fact that with the years there is also an increase of humidity in oil as well as an increase of the oxidation byproducts. Trans formers which are in cities or in urban localities do not usually suffer lightning strokes, consequently the only stresses come from switching overvoltages and/or some high currents.

Fig. 3: Chromometer Hellige Comparator

Fig. 4: Typical glass colored disc

As the years pass by, humidity increases because fluctuations in temperature cause an increase of humidity entering the main oil volume. In distribution transformers having silica gel, the latter looses its absorbing property with an increasing number of years in service. Consequently silica gel should be replaced at regular intervals. The increasing number of years in service influences also the oil oxidation. Oxidation byproducts in turn affect tanδ. The loss factor, however, is not influenced from the existence of humidity in the samples and for this reason is not a criterion for its existence. Interfacial tension is reduced with the years in service. This is due to the increasing quantity of humidity as well as to the byproducts of oxidation. There is, however, a number of samples with rather reduced interfacial tension although the number of years in service was not that big. This may be due to the quality of the oil used or to the fact that it contained only a small quantity of anti-oxidants. Finally, the oil color changes with the years in service. A change of color may also be due to the overcharging of a transformer.

Discussion

In this work, sampling from 50 transformers of 20/0.4 kV was carried out. Sampling and measurements were performed as suggested by the various international specifications [7]. All transformers were in the major Athens area. It should be noted that the transformers were not all from the same manufacturer. There were transformers from various manufacturers, both from Greece and abroad. This is one of the complications of the Greek Electricity System and certainly, this fact hinders any thought for a statistical approach. In case of poor oil color, this may be a symptom of humidity. Poor oil dielectric strength may imply that the oil should be cleaned or replaced. If the humidity level is high, the oil has to be again either cleaned or replaced. If the interfacial tension of oil is acceptable, this means that a control for the possible presence of byproducts may be performed. Acceptable values of oil resistivity may be tolerated. In most cases, tanδ was good. The parameters which changed most were the color, the oil dielectric strength, the levels of humidity and the oil interfacial tension. From the above, it is evident that one cannot pronounce any verdict on the quality of a transformer oil based on only one parameter.

It is evident from the results that there was absence of a transpiring system with silica gel in most investigated transformers. This absence has as a result that the humidity of the atmosphere is in contact with the oil. The functioning of a transformer under heavy load, and consequently its functioning at high temperatures, results in an acceleration of its ageing. High temperatures cause oxidation of the oil. In addition to that, in the last years, increased loads – as a consequence of climatic changes (such as increased temperature and humidity) – are required and they result in the stressing of the oil. Arcs, because of short circuits, create gaseous byproducts and sludge. The latter influence in a negative way the dielectric strength of the oil. Another point which should be considered is that, with the continuing effort from the side of the manufacturers for the reduction in the size of the transformers, the oil tends to be more easily thermally stressed. This results in an accelerated rate of rise in temperature and an accelerated rate in its ageing.

Generally speaking, the frequent oil sampling helps in pointing out potential trouble spots in the network. It goes without say that, as in this paper, a variety of measurements may give a better picture of the actual state of the transformers rather than isolated measurements or measurements confined only on a single quantity (e.g. dielectric strength). Ageing is a complex process and, as is well known, a variety of factors play a role. Ageing factors contribute not only to the ageing of the oil, but they may also interact among themselves, rendering thus the prediction of lifetime even more difficult [8], [9]. Breakdown of transformer oil is also a complex phenomenon determined mostly by the operating conditions [10].

As a way out of the various problems with transformer ageing, the construction of additional substations is proposed. This will reduce the load of each transformer. Needless to say that, the continuous checking of the whole distribution network and the requirement from the manufacturers to observe all the international norms and specifications, are necessary conditions for the good functioning of the whole system.

Special attention should be paid to the way one gets the oil samples. Sampling should be done with due care and care should be taken in order not for the sample to contain water. The samples should be taken from the transformers when the oil is still warm. Foreign particles should be allowed to settle at the bottom, so that no undue influence can be recorded in the various measurements.

In the case the results are contradictory, one has to repeat the sampling and the corresponding measurements. It is true that the manufacturers, try to reduce the production cost of the transformers and their respective materials by reducing the size of the transformers. This on one hand has as a result the possibly higher break-down strength of the oil, on the other hand, however, has as a result the faster increase of the oil temperature, and consequently the faster ageing and stressing of the mechanical parts of the transformer. Anyhow, a better maintenance of the distribution transformers should be followed as well as the placing of more transformers of larger power in localities with an increasing load demand.

The discussion as well as the conclusions of the present paper are in line with previously published work [11], [12]. Such work was carried out on transformers of 20/0.4 kV as well as of 150/20 kV. A statistical approach cannot be offered for the time being, as it was done before [13], for the reasons given above.

It is to be hoped that the next steps in our work will be a classification of the distribution transformers according to manufacturer, according to their power (kVA) and according to the previous history (which includes years in service, all faults, short-circuits, lightning strokes, switching operations etc.). Thereafter, an effort will be made in order to statistically analyze the data.

Concluding, it is fitting to note that the present paper does not claim any originality regarding any new diagnostic techniques. This paper is an application of already existing and well known specifications. It is a continuation of an effort started few years ago, as mentioned in [7], [11], [12]. In this respect, the present paper presents new results, although the methods used are the same as in [6], [7], [11], [12]. In other words, this paper presents the state of transformer oil in some distribution transformers of a given area at a given moment in time. Needless to say that continuous monitoring is necessary in order to check and confirm the transformer oil quality. Continuous monitoring, however, is often not enough. If possible, it should go together with fault detection, namely partial discharge (PD) detection. PD detection can help preventing a defect from developing into a fault, e.g. a short circuit [14], [15], [16]. Nevertheless, condition monitoring as presented here, should be done on a regular basis.

Conclusion

Transformers of 20/0.4 kV from the major Athens area were investigated. Various parameters (oil color, dielectric strength, humidity, interfacial tension, tanδ and resistivity) of the oil were studied with standard methods based on international specifications. No verdict on transformer oil quality can be based on only one parameter. A multitude of parameters is needed in order to pronounce a correct verdict on oil quality. The continuous control and monitoring of the distribution transformers is necessary in order to avoid problematic situations. The role of silica gel is emphasized.

Silica gel should be used in all transformers, so that humidity can be absorbed. The quantity of humidity increases with the number of years in service, consequently, distribution transformers should be checked at regular intervals.

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* About The Authors

Michael Danikas, Issue 2, p. 39

Elli Rapti, is a graduate of Democritus University of Thrace, Department of Electrical and Computer Engineering, Xanti, Greece. She is currently employed in a private-owned company. Her research interests are in transformer oil diagnostics.

Ioannis Liapis, is a graduate of Democritus University of Thrace, Department of Electrical and Computer Engineering, Xanthi, Greece, and is currently working with Public Power Corporation in Athens. His research interests are in transformer oil diagnostic techniques.
j-liapis@hotmail.com

Ahmad Basri Bin Abdul Ghani, Dr., is a researcher at TNB Research Institute in Selangor, Malaysia. His research interests include among other topics, power cable diagnostics, transformer oil diagnostics and partial discharge measurements.
abasri.aghani@tnbr.com.my

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FRONT COVER VIGNETTE

A faded synthesis of an anthemion rooted in a meandros

The thirteen-leaf is a symbol for a life tree leaf.
"Herakles and Kerberos", ca. 530–500 BC,
by Paseas, the Kerberos Painter,
Museum of Fine Arts, Boston.

www.mfa.org/collections/object/plate-153852

The simple meandros is a symbol for eternal immortality.
"Warrior with a phiale", ca. 480–460 BC,
by Berliner Maler,
Museo Archeologico Regionale "Antonio Salinas" di Palermo.

commons.wikimedia.org/wiki/File:Warrior_MAR_Palermo_NI2134.jpg