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# Alternative Insulating Gases to SF<sub>6</sub>: A Short Review

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## Abstract

The present paper undertakes the study of sulfur hexafluoride (SF<sub>6</sub>) as an insulating gas. SF<sub>6</sub> is a modern technology gas, with the extraordinary ability to instantly "extinguish" electric arcs, thus it is mainly used in the equipment of electricity generation, transmission and distribution grid, but it can also be used in a multitude of other kinds of applications. Despite its many advantages, the use of SF<sub>6</sub> also comes with some serious negative effects and it is important to be restricted. For instance, it is the gas that contributes the most to the "greenhouse effect" and it has a significant impact on global climate change. After the presentation of the gas and the applications in which it is used, reference is being made to the problems caused by its use and there will be a presentation of some other insulating gases that can be used as alternative solutions.

## Keywords

Insulating gas, sulfur hexafluoride, electrical grid, circuit breaker, greenhouse effect, toxic by-products, alternative gases

## Introduction

Sulfur hexafluoride (SF<sub>6</sub>) is an electronegative gas which is appropriate for switchgear. Its ability to extinguish arcs is remarkable and

this is the reason why it is widely used in the high voltage industry. More precisely, SF<sub>6</sub> used in substations, in gas insulated switchgear (GIS), in high voltage cables as

well as insulating gas in electrostatic machines [1]. It was discovered by the Nobel prize winner in Chemistry Henri Moissan [2]. Its synthesis is obtained with the reaction of F with S. During this reaction other substances in smaller quantities are also obtained, such as  $S_2$ ,  $F_{10}$  and  $SF_4$ . Such substances, in contradistinction to  $SF_6$ , are toxic and they must be removed. Sulfur hexafluoride can be stable without any decomposition in its molecular structure up to  $500^\circ C$ , it is not flammable and it does not react with  $H_2O$  or  $Cl$ . The density of  $SF_6$  is about  $6kg / m^3$  under normal temperature and pressure and this renders the aforementioned gas five (5) times heavier than air. Its specific thermal conductivity is three (3) times higher than that of the air. Consequently,  $SF_6$  has an excellent cooling capability. Sulfur hexafluoride is under normal conditions non-toxic, chemically inert and stable. Both S and F are electronegative chemical elements and subsequently free electrons are attached to  $SF_6$  creating thus negative ions  $SF_6^-$ . Such ions are heavy, moving slowly and thus they render ionisation more difficult, increasing in this way the breakdown strength of  $SF_6$ . In other words, the molecules of  $SF_6$  render the phenomenon of elec-

tron avalanche slower. The dielectric strength of  $SF_6$  is three times higher than that of the air under normal pressure [3].

Most of the byproducts - after a breakdown or arcing - do not degrade the dielectric strength of  $SF_6$  and can be easily removed. During arcing there is no polymerisation with carbon or other conducting byproducts.  $SF_6$  is compatible with most insulating and conducting materials. Its dielectric constant at 1.0133 bar is 1.0021 at  $200^\circ C$ , whereas with a pressure of 20 bar the dielectric constant increases by 6%. The dielectric strength of  $SF_6$  follows Paschen's law (i.e. with the breakdown voltage being dependent on the function of  $pd$ , where  $p$  is the gas pressure and  $d$  is the gap spacing in a homogeneous electric field) [4, 5]. Another advantage of  $SF_6$  is its cooling ability with its time for the extinguishing of an arc being 100 times lower than that of air.

The usefulness of  $SF_6$  is not limited to the electrical industry. In the metallurgical industry  $SF_6$  acts as protective inert gas in order to prevent re-ignition of magnesium, to help in removing pollution and gases such as hydrogen ( $H_2$ ) in the case of aluminium production, to the



production of semiconductors (dry etching), in medicine (eye surgery as well as in ultrasound applications), in housing and in oceanography.

Regarding the electrical industry, 80% of SF<sub>6</sub> is used for the generation and transmission of electrical power. Its wide use includes switchgear (GIS) up to several hundreds of kV since it prevents the formation of arcing [6, 7]. Its thermal conductivity together with its speedy recovery of dielectric strength after arcing renders it suitable for switching devices. Such properties allow the size reduction of GIS in comparison to AIS. Its electronegativity gives the SF<sub>6</sub> GIS the capability of absorbing the energy of the electrons in case of arcing and thus to decrease the temperature of the arc. SF<sub>6</sub> can also recover easily and quickly its dielectric strength, consequently it is an ideal medium for quenching the arc. The total space required for a GIS with SF<sub>6</sub> is only a fraction of that required by a GIS with air [3, 4, 8].

SF<sub>6</sub> is also used in Gas Insulated Transmission Lines (GITL), where it is used, among others, as insulating medium in high density populated industrial areas. GITL offer high transmission ability, low losses, low capaci-

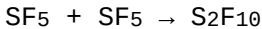
tance, low external magnetic field, non-flammability, reliability, compact solutions in case of high density areas, no interference in telecommunication systems. On the other hand, some disadvantages are the high cost of such a line, the limited length, the danger of pollution from particles which diminishes the insulating capability. The cost at the moment is 6 to 8 times higher than that of conventional lines. A solution to that is the replacement of SF<sub>6</sub> by a mixture of SF<sub>6</sub> and nitrogen, which is cheaper [9]. GITL are used mainly for shorter transmission lengths since they can be deformed because of the change in volume (and consequently of pressure). An alternative is also the replacement of SF<sub>6</sub> with air of high quality and high pressure.

SF<sub>6</sub> is also used in transformers, where it may replace the oil. Because of its non-flammability it is preferred (besides its high dielectric strength, its compatibility with solid insulation and its cooling capability). In transformers, the lower noise, the lower cost of maintenance, the higher expected lifetime, the high reliability, are the significant advantages. On the other hand, its effect on the environment and its lesser

thermal capacity than the oil, are distinct disadvantages [10 - 12].

### Problems in the use of SF<sub>6</sub>

When an electric discharge occurs a part of SF<sub>6</sub> decomposes in lower chemical substances which in turn give some chemical byproducts. The probable formation of byproducts such as SF<sub>4</sub>, SF<sub>2</sub>, S<sub>2</sub>F<sub>10</sub>, SO<sub>2</sub>, S<sub>2</sub>O<sub>2</sub>F<sub>10</sub>, HF, SOF<sub>2</sub>, SOF<sub>4</sub>, SO<sub>2</sub>F<sub>2</sub>, SOF<sub>10</sub>, and H<sub>2</sub>S is well established. Some of these byproducts are toxic, e.g. S<sub>2</sub>F<sub>10</sub>. The latter is due to the reaction



where, SF<sub>5</sub> is formed from the decomposition of SF<sub>6</sub> as a result of the collision with electrons. Minute quantities of SO<sub>2</sub>F<sub>2</sub> and S<sub>2</sub>F<sub>10</sub> may cause severe health problems to humans [13].

Furthermore, byproducts of SF<sub>6</sub> are related to the greenhouse effect. Greenhouse gases are gases that absorb part of infrared radiation from the earth and they return it to the earth. Such gases appear either in the natural environment (e.g. H<sub>2</sub>O, CO<sub>2</sub>, CH<sub>4</sub>, N<sub>2</sub>O) or they are artificial products such as SF<sub>6</sub> and products of fluorinated compounds (FFC) as well as reaction products such as CO<sub>2</sub>, nitrogen, and sulphur oxides.

The trapping of infrared radiation from the gases and the re-emission rises the earth temperature. The re-emission of infrared radiation back to earth has as consequence the rise of temperature. One of the man-made re-absorptions is leakage of SF<sub>6</sub> from substations and from the metal industry [14].

In a period of 100 years, SF<sub>6</sub> is 23000 times more efficient in trapping infrared radiation than an equivalent quantity of carbon dioxide (CO<sub>2</sub>). SF<sub>6</sub> may be emitted and accumulated in the atmosphere and may affect the climate change for many centuries (the lifetime of SF<sub>6</sub> in the atmosphere is more than 3000 years). Since it is not easily decomposed, its contribution to the rise of temperature of the earth may be cumulative and permanent. Leaks of SF<sub>6</sub> may come from the electrical industry, from substations, from testing of electrical equipment etc. [15 - 18].

### The situation today - Alternative proposals

Substations and related systems continue to use SF<sub>6</sub>. Researchers from Bristol confirmed significant increases of the said gas since 1995 from 3.5 ppt per trillion to 10.5 ppt per trillion in 2021 [17]. It is true that SF<sub>6</sub>

concentration is still small compared with that of CO<sub>2</sub>. Its concentration, however, is expected to increase until the year 2030. Another worrying aspect is that SF<sub>6</sub> is a gas that cannot be decomposed or destroyed by physical means. The fact that SF<sub>6</sub> is still be used is that there are very strong reactions from the electrical industry. Another reason is that there are not still in sight alternative solutions to the use of the SF<sub>6</sub>.

SF<sub>6</sub> was first used as insulating medium for switchgear in 1938 by Grosse. Westinghouse Electric Co. was the first to manufacture SF<sub>6</sub> switchgear for the 115 kV network [19]. From then on the said gas was used in various types of electrical equipment as insulating medium. It is still a widely used gas because of the properties mentioned above. As alternatives to SF<sub>6</sub>, mixtures of the said gas with nitrogen (with 50%-60% SF<sub>6</sub>) were used. The dielectric strength of the mixture may reach 85-90% of the dielectric strength of the pure SF<sub>6</sub>. It was shown that an 800 kV transmission line using this mixture costs only 21% of the cost when pure SF<sub>6</sub> is used. Mixtures of SF<sub>6</sub> with air, N<sub>2</sub>O, N<sub>2</sub>, CO<sub>2</sub> were also tried [20]. A transmission line with a mixture of SF<sub>6</sub>

/N<sub>2</sub> was used in the beginning of this century in Geneva, Switzerland. Efforts were also made to use the above mixture in a 420 kV line in France, where the percentage SF<sub>6</sub> in the mixture was reduced by 30% [21]. Such mixtures are also tried in DC systems, the main counterargument being that in DC conditions metallic particles may play in even more crucial role in the degradation.

Such a mixture of SF<sub>6</sub> with other gases may well offer a solution w.r.t. the satisfactory functioning of electrical equipment and may reduce the use of SF<sub>6</sub> to a significant degree but its use cannot be totally excluded. Consequently, various researchers tried to propose gases with similar characteristics which may not be detrimental to the environment. Given the electronegativity of SF<sub>6</sub> efforts were made so that the alternative gases preserve this characteristic without the toxicity.

Devins studied the breakdown voltage of several electronegative gases, such as CF<sub>4</sub>, C<sub>3</sub>F<sub>8</sub>, C<sub>4</sub>F<sub>10</sub> and C<sub>2</sub>F<sub>6</sub>. These fluorocarbons are stable and electronegative. Regarding their insulating properties the above gases are classified with the following order C<sub>6</sub>F<sub>14</sub> > C<sub>4</sub>F<sub>10</sub> > C<sub>3</sub>F<sub>8</sub> > C<sub>2</sub>F<sub>6</sub> > CF<sub>4</sub> [22]. The problem

with the above gases is that they are also included in the gases whose use must be limited according to Kyoto protocol because of their global warming potential (GWP).

The gas  $\text{CF}_2\text{Cl}_2$  has dielectric strength similar to that of  $\text{SF}_6$  whereas the mixture  $\text{CF}_2\text{Cl}_2\text{-CO}_2$  has insulating properties similar to  $\text{CF}_2\text{Cl}_2\text{-N}_2$  but it differs considerably from the mixture  $\text{SF}_6\text{-CO}_2$  [21]. Towards the end of the nineties with the greenhouse effect to having worsened, attention was paid to perfluorocarbons (PFC) and hydrofluorocarbons (HFC) because of their remarkable insulating properties [23]. By the end of the last century but also earlier, the physico-chemical and insulating properties of the pure c-C<sub>4</sub>F<sub>8</sub> were studied. With a uniform electric field, the aforementioned gas presented 1.18 to 1.25 times higher dielectric strength than  $\text{SF}_6$ . Such a mixture, however, cannot be used in high altitudes because of its high temperature of liquefaction [24]. Research efforts were made with other combinations, such as c-C<sub>4</sub>F<sub>8</sub>/N<sub>2</sub>, c-C<sub>4</sub>F<sub>8</sub>/air, and c-C<sub>4</sub>F<sub>8</sub>/CO<sub>2</sub> regarding their insulating properties thus finding that they have higher dielectric strength than the mixture  $\text{SF}_6/\text{N}_2$  with a uniform electrode arrangement [25].

Yet research on c-C<sub>4</sub>F<sub>8</sub> with gases such as N<sub>2</sub>, CO<sub>2</sub>, N<sub>2</sub>O, CHF<sub>3</sub> and CF<sub>4</sub> revealed that the dielectric strength of the mixtures c-C<sub>4</sub>F<sub>8</sub>/N<sub>2</sub>, c-C<sub>4</sub>F<sub>8</sub>/CO<sub>2</sub> and c-C<sub>4</sub>F<sub>8</sub>/CF<sub>4</sub> increases linearly with increasing percentage of C<sub>4</sub>F<sub>8</sub>. The best mixture analogies of the c-C<sub>4</sub>F<sub>8</sub>/CO<sub>2</sub> and c-C<sub>4</sub>F<sub>8</sub>/N<sub>2</sub> are 10% and 20% respectively regarding the AC dielectric strength. With respect to the insulating properties the classification of the above gases is

$$\begin{aligned} \text{c-C}_4\text{F}_8/\text{N}_2 &> \text{c-C}_4\text{F}_8/\text{CHF}_3 > \\ \text{c-C}_4\text{F}_8/\text{CO}_2 &> \text{c-C}_4\text{F}_8/\text{CF}_4 \end{aligned}$$

with the remark that c-C<sub>4</sub>F<sub>8</sub> renders sedimentation of the carbon atoms unavoidable during discharge diminishing thus the insulating properties of the mixture [26]. Work was done on c-C<sub>4</sub>F<sub>8</sub>/N<sub>2</sub> with different pressures and different electrode distances. The inception voltage for the pure c-C<sub>4</sub>F<sub>8</sub> is about 1.3 times higher than that of  $\text{SF}_6$ . c-C<sub>4</sub>F<sub>8</sub> and N<sub>2</sub> show that they have a sort of synergistic effect when mixed together [27].

Relatively recently it was shown that in tests with switchgear of 145 kV a mixture of C<sub>4</sub>F<sub>7</sub>N/CO<sub>2</sub> in analogy of 18-20% could obtain the same dielectric strength as the pure  $\text{SF}_6$ . Experiments with AC voltages as well with im-

pulse voltages showed that the above mixture presented satisfactory electrical behavior. An important point that was emphasized is that the above data are valid for uniform and quasi-uniform electric fields but not for non-uniform fields. This implies that attention must be paid to the equipment design so that non-uniform fields are avoided since the mixture of fluoronitrile/CO<sub>2</sub> tends to give lower dielectric strength than SF<sub>6</sub> with non-uniform fields [28].

The gases C<sub>4</sub>F<sub>8</sub> and C<sub>3</sub>F<sub>8</sub> are not typical greenhouse gases, however, their GWP is rather high (8700 and 7000 respectively) and they can stay in the atmosphere for a very long time (3200 and 2600 years respectively) [23].

Novec 5110 - C<sub>5</sub>F<sub>10</sub>O and Novec 4710 - C<sub>4</sub>F<sub>7</sub>N are also alternatives to SF<sub>6</sub>. These are high density gases that are non-flammable and they do not destroy the ozone of the atmosphere. Their dielectric strength is superior to that of SF<sub>6</sub> and their GWP is much lower to that of the SF<sub>6</sub> [29, 30]. Their boiling point is much higher than that of SF<sub>6</sub>, which means that their pressure is lower at any temperature. They can cause much lesser damage to the environment since they remain in the atmosphere 0.4 and 30 years

respectively. They are much less damaging for the environment since their contribution to the greenhouse effect is minimal. When the above gases are mixed with air, their dielectric strength gets higher when the percentage of the gases in the mixture is higher. It must, however, be noted that for the same pressure the mixtures of the aforementioned gases with air, their dielectric strength is lower than that of the pure SF<sub>6</sub>. The mixtures can be used at higher pressures (in the range of 5.2 bar) in order to reach the dielectric strength of the pure SF<sub>6</sub>. Both C<sub>5</sub>F<sub>10</sub>O and C<sub>4</sub>F<sub>7</sub>N have a better electronegativity than SF<sub>6</sub> and their thermal properties are satisfactory and from a medical point of view are safe [31].

Regarding another possible replacement of SF<sub>6</sub> researchers proposed CF<sub>3</sub>I, which is a gas which also captures electrons. It is colorless and non-flammable. Its environmental effects are negligible. It has, however, a relatively high temperature of liquefaction (-22.5° C) which implies that for wider use it has to be mixed with other gases with a lower temperature of liquefaction. The main by-products after discharge activity are C<sub>2</sub>F<sub>6</sub>, C<sub>2</sub>F<sub>4</sub>, C<sub>2</sub>F<sub>5</sub>I, C<sub>3</sub>F<sub>8</sub>, CHF<sub>3</sub>, C<sub>3</sub>F<sub>6</sub> and CH<sub>3</sub>I with

the first two to be the main decomposition byproducts, with  $C_2F_6$  to be the main by-product independently of whether the applied electric field is uniform or non-uniform [32]. In an earlier investigation, the same authors noted that the V-t characteristics of  $CF_3I$ ,  $SF_6$ ,  $CF_3I/N_2$  and  $SF_6/N_2$  gas mixtures under non-uniform field gaps by using the steep-front square pulse voltage showed that with a more uniform electric field the sparkover voltage of  $CF_3I$  is higher than that of  $SF_6$  whereas a  $CF_3I/N_2$  gas mixture containing  $N_2$  gas of 40% are equivalent to those in pure  $SF_6$  gas as far as the V-t characteristics are concerned for uniform electric field [33].  $CF_3I$  has low toxicity and does not cause any damage to the immune system of humans. On the other hand,  $C_3F_8$  as a by-product of discharge activity can cause weakness and problems in sleep but it exists only in minute quantities. In case of a surface flashover, the concentration of  $C_3F_8$  is only 0.00122 ppm under uniform field conditions and only 0.000501 ppm under non-uniform electric field. With a rod-plane electrode arrangement  $CF_3I$  has generally a higher dielectric strength than  $SF_6$  for higher pressures whereas the relation is reversed at lower pressures [21] and with nonuni-

form electrodes. With lightning voltages, the dielectric strength of pure  $CF_3I$  is higher than that of  $SF_6$  for spherical electrodes. With a mixture of  $CF_3I/CO_2$ , having 60% of  $CF_3I$ , the dielectric strength may reach the dielectric strength of the pure  $SF_6$ . The percentage increase of  $CF_3I$  in mixture with either  $CO_2$  or  $N_2$  results in an increase of the dielectric strength [34]. In a mixture of 30%/70%  $CF_3I/CO_2$  the dielectric strength increases with the electrode uniformity, whereas the insulating properties of 30%/70%  $CF_3I/CO_2$  are similar with those of the mixture 20%/80%  $SF_6/N_2$  [18, 35]. Yet other work indicated that with a 60% percentage of  $CF_3I$  the V-t characteristic of  $CF_3I/N_2$  or of  $CF_3I/air$  the dielectric strength is similar to that of  $SF_6$  [36]. Research performed on mixed gas  $C_4F_7N/CO_2$  indicated a dielectric strength comparable or even better than that of  $SF_6$ , drawing also attention to the faster decomposition rate of the former with increasing temperature [37].

Regarding the inception voltage of partial discharges with a point/plane electrode arrangement, it was found that this does not vary either with pure  $CF_3I$  or pure  $SF_6$  at 0.1 Mpa. A mixture of  $CF_3I/CO_2$

presents an inception voltage by about 0.9 - 1.1 times higher than that of a mixture of SF<sub>6</sub>/CO<sub>2</sub> [20]. When the volume of CF<sub>3</sub>I is about 20% in a mixture of CF<sub>3</sub>I/N<sub>2</sub>, the inception voltage was 0.92 - 0.94 times higher than that of a mixture SF<sub>6</sub>/N<sub>2</sub> provided that the experimental conditions are the same [20]. With respect to the arc quenching, both the pure CF<sub>3</sub>I and the mixture CF<sub>3</sub>I/CO<sub>2</sub> present a satisfactory performance compared to SF<sub>6</sub> in the case of Short Line Fault (SLF) and in the case of Breaker Terminal Fault (BTF) [20].

Regarding modern applications of the above, one may emphasize that efforts are being made in several countries. Combinations of C<sub>5</sub>F<sub>10</sub>O with dry air and CO<sub>2</sub> have been tried in RMU of 24 kV with a nominal current of 630 A with satisfactory results.

Synthetic air in 12 - 36 kV/1250 A equipment is also being tried [38]. GIS with C<sub>5</sub>F<sub>10</sub>O is also in use in Switzerland [39].

### Conclusion

Since even a small quantity of SF<sub>6</sub> may have detrimental effects in climate change, alternatives were being sought. Today's alternatives suggest other gases as replacements. There are, however, some hindrances either because research is - and rightly so - slow or because industry seems to be even slower to react. Alternatives gases to the SF<sub>6</sub> and/or mixtures of gases were presented and in all probability these seem to be viable alternatives. However, a lot will depend on the approach of the various governments to the problem of climate change.

### References

- [1] Kuffel E., Zaengl W. S., Kuffel J., "High Voltage Engineering Fundamentals", Eds. Newnes, Oxford, UK, 2000, p. 306, pp. 345 - 348
- [2] Amato Ivan, "1906 Chemistry nobelist Henri Moissan spawned the vast arena of fluorine chemistry", Chemical and Engineering News, Vol. 84, Issue 47, 2006 <https://cen.acs.org/articles/84/i47/1906-Chemistry-Nobelists-Henri-Moissan.html>
- [3] Gallagher T. J., Pearmain A. J., "High Voltage: Measurement, Testing, and design", Eds. John Wiley & Sons, New York, USA, 1983, p. 12, pp. 46 - 47, p. 118, p. 203

- [4] Kind D., Kaerner H., "High-Voltage Insulation Technology", EDds. Vieweg & Sohn, Braunschweig Germany, 1985, pp. 72 - 74, p. 147, p. 158
- [5] Wadhwa C. L., "High Voltage Engineering", Eds. New Age International (P) Ltd., Publishers, New Delhi, India, 2001, p. 15
- [6] Billen P., Maes B., Larrain M., Braet J., "Replacing SF<sub>6</sub> in electrical gas-insulated switchgear: Technological alternatives, and potential life cycle greenhouse gas savings in an EU-28 perspective", Energies, Vol. 13, No. 7, 2020, <https://doi.org/10.3390/en13071807>
- [7] Koch D., "SF<sub>6</sub> properties, and use in MV and HV switchgear", Cahier technique No. 188, Schneider Electric, 2003, 22 pages, [https://studiecd.dk/cahiers\\_techniques/SF6\\_properties.pdf](https://studiecd.dk/cahiers_techniques/SF6_properties.pdf)
- [8] Naidu M. S., Maller V. N., "SF<sub>6</sub> and vacuum insulation for high voltage applications", Eds. Khanna Publishers, New Delhi, India, 1998
- [9] Koch H., "Experience with 2<sup>nd</sup> generation gas-insulated transmission lines GIL", Proceedings of JICABLE, <https://www.jicable.org/wets03/pdf/wets03-1-06.pdf>
- [10] Walsh E. J., Robison J. A., Wendel R. C., "SF<sub>6</sub>/oil dielectric for power transformers", EPRI Final Report, EPRI-EL-1358, Research Project 808-1, March 1980 <https://www.osti.gov/servlets/purl/5319131>
- [11] [https://www.gegridsolutions.com/products/brochures/xdge/currenttransformers\\_iec\\_print.pdf](https://www.gegridsolutions.com/products/brochures/xdge/currenttransformers_iec_print.pdf)
- [12] <https://www.think-grid.org/search-sf6-replacement>
- [13] U.S. Environmental Protection Agency (Office of Air and Radiation Global Programs Division), "Byproducts of sulphur hexafluoride (SF<sub>6</sub>) - Use in the electric power industry", January 2002, [https://www.epa.gov/system/files/documents/2022-05/sf6\\_byproducts.pdf](https://www.epa.gov/system/files/documents/2022-05/sf6_byproducts.pdf)
- [14] <https://www.britannica.com/event/Kyoto-Protocol>
- [15] Pan B., Wang G., Shi H., Shen J., Ji H.-K., Kil G.S., "Green gas for grid as an eco friendly alternative insulation gas to SF<sub>6</sub>: A review", Applied Sciences, Vol. 10, 2020, 13 pages, doi:10.3390/app10072526



- [16] Abdel-Salam M., "High-Voltage Engineering: Theory and Practice", 2<sup>nd</sup> edition, CRC Press, Boca Raton, USA, 2000, pp. 261 - 341
- [17] McGrath M., "Climate change: Electrical industry's 'dirty secret' boosts warming", 13 September 2019, <https://www.bbc.com/news/science-environment-49567197>
- [18] Widger P., Haddad A., "Evaluation of SF<sub>6</sub> leakage from gas insulated equipment on electricity networks in Great Britain", *Energies*, Vol. 11, No. 8, 2018, <https://www.mdpi.com/1996-1073/11/8/2037>
- [19] <https://www.mettz.com/products/sf6-breakers>
- [20] Rokunohe T., Yagihashi Y., Endo F., Oomori T., "Fundamental insulation characteristics of air, N<sub>2</sub>, CO<sub>2</sub>, N<sub>2</sub>/O<sub>2</sub>, and SF<sub>6</sub>/N<sub>2</sub> mixed gases", *Electrical Engineering in Japan*, Vol. 155, No. 3, 2006, pp. 9 - 17
- [21] Xiao S., Zhang X., Tang J., Liu S., "A review on SF<sub>6</sub> substitute gases and research status of CF<sub>3</sub>I gases", *Energy Reports*, Vol. 4, 2018, pp. 486 - 496
- [22] Devins J. C., "Replacement Gases for SF<sub>6</sub>", *IEEE Transactions on Electrical Insulation*, Vol. 15, No. 2, 1980, pp. 81 - 86
- [23] Duzkaya H., Tezcan S. S., Acartürk A., Yilmaz M., "Environmental and physiochemical properties of gaseous dielectrics alternatives to SF<sub>6</sub>", *El-Cezerî Journal of Science and Engineering*, Vol.7, No: 3, 2020, pp. 1460 - 1470
- [24] Qiu Y., Kuffel E., "Dielectric strength of gas mixtures comprising SF<sub>6</sub>, CO, c-C<sub>4</sub>-F<sub>8</sub> and SF<sub>6</sub>, N<sub>2</sub>, c-C<sub>4</sub>F<sub>8</sub>", *IEEE Transactions on Power Apparatus and Systems*, Vol. 102, No. 5, 1983, pp. 1445 - 1451
- [25] Zhao H., Li X., Lin H., "Insulation characteristics of c-C<sub>4</sub>-F<sub>8</sub>-N<sub>2</sub> and CF<sub>3</sub>I-N<sub>2</sub> mixtures as possible substitutes for SF<sub>6</sub>", *IEEE Transactions on Power Delivery*, Vol. 12, No. 1, 2017, pp. 254 - 262
- [26] Li X., Zhao H., Jia S., Murphy A. B., "Prediction of the dielectric strength for c-C<sub>4</sub> F<sub>8</sub> mixtures with CF<sub>4</sub>, CO<sub>2</sub>, N<sub>2</sub>, O<sub>2</sub> and air by Boltzmann equation analysis", *Journal of Physics D: Applied Physics*, Vol. 47, 2014, 9 pages <https://iopscience.iop.org/article/10.1088/0022-3727/47/42/425204/pdf>

- [27] Zhao S., Xiao D., "Research progress on synergistic effect between insulation gas mixtures", book chapter from the book "Modern Applications of Electrostatics and Dielectrics", edited by D. Xiao and K. Sankaran, DOI 10.5772/intechopen.90705, <https://www.intechopen.com/chapters/70707>
- [28] Zhang B., Uzelac N., Cao Y., "Fluoronitrile/CO<sub>2</sub> mixture as an eco-friendly alternative to SF<sub>6</sub> for medium voltage switchgear", IEEE Transactions on Dielectrics and Electrical Insulation, Vol. 25, No. 4, 2018, pp. 1340 - 1350
- [29] Owens J., Xiao A., Bonk J., DeLorme M., Zhang A., "Recent development of two Alternative gases to SF for high voltage electrical power applications", Energies, Vol. 14, No. 16, 2021, <https://doi.org/10.3390/en14165051>
- [30] <https://electra.cigre.org/310-june-2020/technical-brochures/application-of-non-sf6-gases-or-gas-mixtures-in-medium-voltage-and-high-voltage-gas-insulated-switchgear.html>
- [31] Xiao A., Owens J. G., Bonk J., Zhang A., Wong C., Tu Y., "Environmentally friendly insulating gases as SF<sub>6</sub> alternatives for power utilities", Proceedings of the 2nd International Conference on Electrical Materials and Power Equipment, Guangzhou, China, 2019, pp. 42 - 48
- [32] T. Takeda, S. Matsuoka, A. Kumada, K. Hidaka, "By-product generation through electrical discharge in CF<sub>3</sub>I gas and its effect to insulation characteristics", IEEE Transactions on Power and Energy, Vol. 131, No. 10, 2011, pp. 859 - 864 <http://dx.doi.org/10.1541/ieejpes.131.859>
- [33] Takeda T., Matsuoka S., Kumada A., Hidaka K., "Spark-over characteristics in CF<sub>3</sub>I gas and CF<sub>3</sub>I/N gas mixture under non-uniform field gaps", IEEE Transactions on Power and Energy, Vol. 130, No. 9, 2010, pp. 813 - 818 <http://dx.doi.org/10.1541/ieejpes.130.813>
- [34] Katagiri H., Kasuya H., Mizoguchi H., Satoru Y., "Investigation of the performance of CF<sub>3</sub>I gas as a possible substitute of SF<sub>6</sub>", IEEE Transactions on Dielectrics and Electrical Insulation, Vol. 15, No. 5, 2008, pp. 1424 - 1429

- [35] Al-Alawi M., "Characteristics of alternative insulated gases in Gas Insulated Systems", M.Sc. Thesis, Cardiff University, School of Engineering, Department of Electrical and Electronic Engineering, UK, 2019
- [36] Eves M., Kilpatrick D., Edwards P., Report: "A Literature review on SF<sub>6</sub> gas alternatives for use on the distribution network", Final report, Western Power Distribution Innovation, 2018
- [37] Tong D., Zhao Q., Zhang S., Wang C., Cao R., Tu Y., Zheng Z., "Partial discharge decomposition characteristics of C<sub>4</sub>F<sub>7</sub>N/CO<sub>2</sub> gas mixture at different reaction temperatures", Proceedings of 2022 IEEE 5th International Electrical and Energy Conference (CIEEC), 27-29 May 2022, Nangjing, China, pp. 3793 - 3797
- [38] "Gas-insulated compact switchgear SafePlusAir" (Compact medium voltage (MV) gas-insulated switchgear (GIS) for secondary distribution, 12 kV, 630 A)  
<https://new.abb.com/medium-voltage/switchgear/gas-insulated-switchgear/iec-gis-rmu-for-secondary-distribution/iec-gas-insulated-compact-switchgear-safeplus-air>
- [39] "AirPlus™, An alternative to SF<sub>6</sub> as an insulation and switching medium in electrical switchgear",  
[https://library.e.abb.com/public/bf0078a9a6b544279b2c5142a0cbf6fa/68-72%202m540\\_EN\\_72dpi.pdf](https://library.e.abb.com/public/bf0078a9a6b544279b2c5142a0cbf6fa/68-72%202m540_EN_72dpi.pdf)

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# Measurement Uncertainty in Network Analyzers: Differential Error DE Analysis of Error Models Part 7: Remarks on the Uncertainty Notions -from Lab VNA to Toy NanoVNA-

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## Abstract

From A Common User's Point Of View [FACUPOV] any Vector Network Analyzer [VNA] -from the most expensive Laboratory VNA [Lab VNA] to an extremely cheap Do It Your Self [DIY] NanoVNA [Toy NanoVNA]- that can deliver all the values of its Calibration and the Device Under Test [DUT] in each one of the measurement frequencies, subjects to the uniquely existing estimation of its Measurement Uncertainty in that frequency, that is the one which is computed after the Exact Formation of its Complex  $\rho$ -DER Differential Error Region, as well as, of its two Real Differential Error Intervals in polar -rather than rectangular- form:  $\rho$ -magnitude DEI and  $\rho$ -phase DEI. However, due to the fast and wide spread through the Internet of a huge bunch of accompanying instructions regarding these matters, confusion has arisen from some of the concepts and notations commonly in use in VNA literature which reappears now again, as they are given obscurely, ambiguously, or even incorrectly. Accordingly, this paper presents in full details now, the work done by the authors and announced in the past during the meetings of the circle members of Automatic Network Analyser Metrology [ANAMET] technology group of National Physical Laboratory [NPL] so as to isolate among the also observed then misconceptions, those that definitely require reformulation of their expressions and perhaps a broader consensus on the range of their values and among them primarily that of phase or argument. In addition, in order to highlight the unprecedented advantages of the full and correct computation of DERs and DEIs, the following Practical Applications are included: (a) a comparison between several different in concept and form VNA quantities, (b) a counter example based on previous authors' work, (c) a 3D representation of complex  $\rho$ -DERs, and (d) a selected num-

ber of characteristic frames of AVI videos that produced by the authors to show the evolution of the 2-D outline of Complex  $\rho$ -DERs and of its two Real  $\rho$ -DEIs, both rectangular and polar, uncertainty width in terms of frequency, while the internet links to all of these videos are also provided.

## Keywords

microwave measurements, network analyzer, differential error region, differential error interval, calibration

### Introduction

Some useful remarks on the uncertainty notions used in VNA measurements, that we had in mind when we began our research for the systematic errors about 30 years ago, and which came up again after the presentation of the 33rd (Rohde & Schwarz, Fleet, 11 May 2010) and 34th (National Physical Laboratory, Teddington, 21 October 2010) ANAMET meetings and reappeared nowadays are presented here. The unclear and not well defined concepts are mainly due to the fact of introducing the complex numbers to represent the VNA measurements since we measure magnitude and phase. Thus, our first concern is to give a well formed formula for the phase determination of a complex number. Then we examine the information given in relevant literature, old and new, about the phase and its uncertainty, and how it can lead to misunderstandings. A comparison for the magnitude and phase uncertainty is

given, with the help of some AVI videos for two different DUTs, a 50-Ohm dc-resistance box as a well closed DUT and a ground plane antenna as an open DUT, of which some representative and notable frames are illustrated.

The merits of using total differentials to determine the measurement uncertainty are discussed and the size of the problem for a numerical evaluation of  $\Delta S_{ij}$  in one-port and two-port measurements is exposed.

A counter example from an already published authors' work is used in order to demonstrate that a full one-port SLO calibration may not always be considered as a preferable one over that of just a short response S calibration. A 3-D representation of the reflection coefficient together with the DERs as beads around its curve in space is given.

A final remark concerns the use of terms magnitude and amplitude for the value -

size of the involved quantities and the terms argument and phase for the angle. Magnitude is usually used for vectors and amplitude for complex numbers in ac signals or waves, such voltage or current. Argument is used more commonly for complex numbers while phase for sinusoidal functions and waves and it is the most preferable term in general. In the rest of the text the use and meaning both of these two couples are considered from the same view point, although, at least our VNA, definitely use the terms magnitude and phase for what it measures as  $S_{ij}$  "vectors" [1]. However, there is a lot of misleading information on the internet and elsewhere about the definition of these quantities. Typically we mention here the case: "Degrees are almost universally used for the phase angles in sinusoidal functions, as in,  $\sin(\omega t + 30^\circ)$ . (Since  $\omega t$  is in radians, this is a case of mixed units.)".

The first announcement of the present work was a twenty minute presentation in the 35th ANAMET meeting of the National Physical Laboratory [NPL] in 20 October 2011 in Teddington, which is available in

[www.antennas.gr/anamet/35/](http://www.antennas.gr/anamet/35/)

All the AVI videos produced in order to enhance the presentation of the subject are available as FLOSS in the same link, as above.

### On the Notion of Phase

The well-known representation of a complex number  $\dot{z}$ , with the dot above the character  $z$  to clearly signifying its complex nature, in Cartesian and Polar form, is shown in Fig. 1 and defined as

$$\begin{aligned} \dot{z} &= x + i y = z \angle \hat{z}, \\ z &= \sqrt{x^2 + y^2}, \hat{z} \in (-\pi, \pi] \\ x &= z \cos(\hat{z}), y = z \sin(\hat{z}) \end{aligned} \quad (1)$$

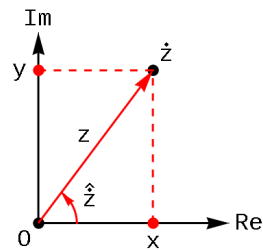


Fig. 1: Complex number

The use of arctangent ( $\tan^{-1}$ ) function to determine  $\hat{z}$  as a function of one variable is inadequate since it returns wrongly the same argument for opposite complex numbers, red and black points shown in the unit circle in Fig. 2, that is, for complex numbers lying in the first and third (I, III) and in the

second and fourth (II, IV) quadrants and returns values only between  $-\pi/2$  and  $+\pi/2$  and not in the whole interval of  $\hat{z}$ , as given in (2),

$$\hat{z} = \tan_p^{-1}\left(\frac{y}{x}\right), \tag{2}$$

$$(-\infty, +\infty) \rightarrow \left(-\frac{\pi}{2}, +\frac{\pi}{2}\right)$$

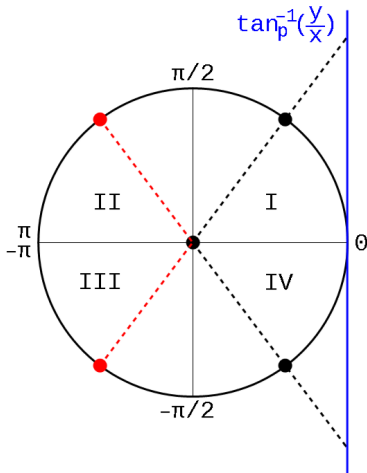


Fig. 2: Arctangent result

It is most appropriate to use the arctangent function of two variables, as defined: a) by taking into account the sign of the real and imaginary part, that is, in which quadrant lies the complex number and b) in terms of the arctangent function of one variable. This function, (3), will return the correct  $\hat{z}$  values in the entire interval

$(-\pi, \pi]$ , as it is expected for complex numbers. Fig. 3 and Tab. 1 give a phase example for nine (9) characteristic points in the unit circle.

$$\text{sgn}(y) = \{y < 0 : -1, y \geq 0 : +1\}$$

$$\hat{z} = \tan_2^{-1}(y, x) = \begin{cases} \tan_p^{-1}\left(\frac{y}{x}\right), & x > 0 \\ \text{sgn}(y) * \pi + \tan_p^{-1}\left(\frac{y}{x}\right), & x < 0 \end{cases} \tag{3}$$

$$= \begin{cases} \text{sgn}(y) * \frac{\pi}{2}, & x = 0 \\ \frac{0}{0}, & x = 0, y = 0 \end{cases}$$

$$(-\infty, +\infty) \times (-\infty, +\infty) \rightarrow (-\pi, +\pi]$$

In Fortran there is the special function ATAN2 and in Mathematica Arg[z] that automatically produce these values, except for the case  $x = 0, y = 0$  of course.

### On the Notion of Phase Uncertainty

1. The first available information on VNA measurements was the very helpful notes of a Vector Seminar by Hewlett-Packard Company [HP] itself in 1989 [2]. As any other printed textbook that can not be changed afterwards, careful study was needed.



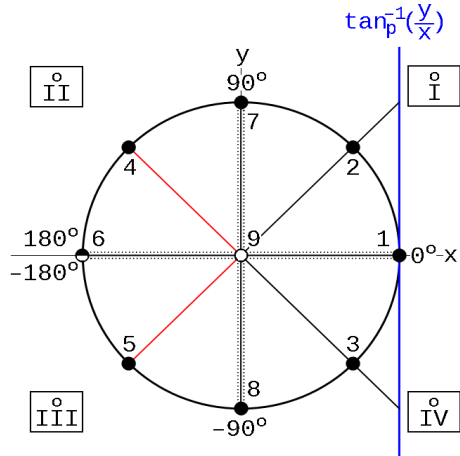


Fig. 3: Phase example - 9 points in unit circle

$$\hat{z} \in (-\pi, -\frac{\pi}{2}) \cup \{-\frac{\pi}{2}\} \cup (-\frac{\pi}{2}, 0) \cup \{0\} \cup (0, \frac{\pi}{2}) \cup \{\frac{\pi}{2}\} \cup (\frac{\pi}{2}, \pi) \cup \{\pi\}$$

Tab. 1: 9 points in unit circle

Point	x	y	$\tan_p^{-1}(y/x)$	$\tan_2^{-1}(y/x)$
1	1	0	0°	0°
2	1	1	45°	45°
3	1	-1	-45°	-45°
4	-1	1	-45°	135°
5	-1	-1	45°	-135°
6	-1	0	0°	180°
7	0	1	$\infty$	90°
8	0	-1	$\infty$	-90°
9	0	0	"0/0"	"0/0"

Thus, in page 3-11 the figure and its adjacent paragraph, shown in Fig. 4, is given, as well as the expressions of  $\Delta\phi$ , as phase uncertainty, inside the slide and of  $\Delta S_{11}$  for the one-port error model of Fig. 5(a), as they appear in the same Seminar notes. Fig. 5(b) shows the one-port error model as it is used in all of our work, with the reflection coefficient  $\rho$  of the DUT as it results after calibration and measurement. Noticeable there is a slightly different notation in the same slide used in the Seminar notes between the figure and the relation below it. However there is an obvious correspondence between the involved quantities as:  $D$  and  $E_D$  of HP with our  $D$ ,  $S_{11M}$  of HP with our  $m$ ,  $M_S$  and  $E_S$  of HP with our  $M$ ,  $1+T_R$ ,  $E_R$  of HP with our  $R$ , and  $S_{11A}$  or  $S_{11a}$  of HP with our  $\rho$ .

Fig. 4 is very interesting since it includes two (2) inconsistencies, one (1) query and one (1) error, as it is explained step-by-step below.

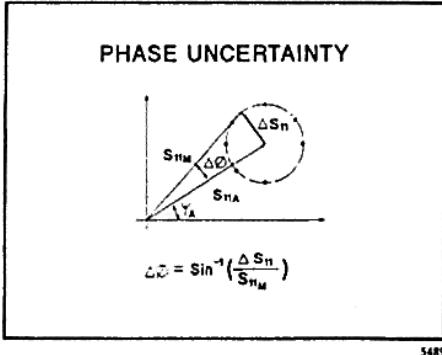
Step 1 - 2 Inconsistencies: In Fig. 6, at the two yellow marked phrases there is a clear reference at  $S_{11}$  after calibration which corresponds to  $S_{11A}$  not to  $S_{11M}$  which is the measured value. But in the  $\Delta\phi$  expression we found the unexpected  $S_{11M}$ .

Step 2 - 1 Query: If " $\Delta S_{11}$  is perpendicular to the value of  $S_{11A}$ ", as it is shown in Fig. 7, then the given expression of arc sinus is correct. The question is: is there any case for this statement to be true, and if it is then when it happens.

Step 3 - 1 Error: In Fig. 8 the statement that this is the worst case is wrong, since the worst case, that is, the one which gives the maximum  $\Delta\phi$ , results only when we consider the tangent to the circle centered at  $S_{11A}$  with radius  $\Delta S_{11}$ . That means, that the worst case for  $\Delta\phi$  will be when  $\Delta S_{11}$  is perpendicular to the  $S_{11M}$  value and not to  $S_{11A}$ . We built the [DELTA PHI. AVI] video in order to reveal the wrong statement. Six frames are shown in Fig. 9, for random points on the circle. All frames contain the actual worst case and the case indicated incorrectly as the worst. The variable is the position around the circle centered at  $S_{11A}$  with radius the value of  $\Delta S_{11}$ . Therefore, for every point on the periphery the  $\Delta\phi$  angle is outlined with blue color and its value is noted in degrees. The angle for the case of Fig. 8, that is, when  $\Delta S_{11}$  is perpendicular to  $S_{11A}$ , as it is stated in the Seminar notes, is sketched out with green color and its value is written, Fig. 8(c).

The actual worst case is outlined with red color in Fig. 8(e), that is, when  $S_{11M}$  is on the tangential line and

there is a  $90^\circ$  angle with  $\Delta S_{11}$ . Since  $28.1^\circ > 25.2^\circ$  the statement under discussion is obviously wrong.

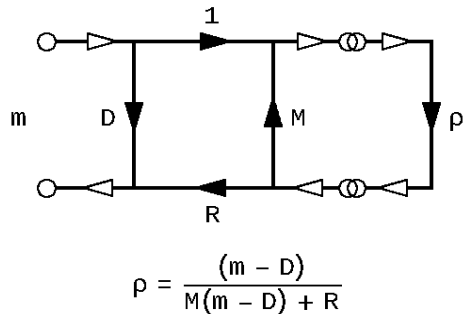
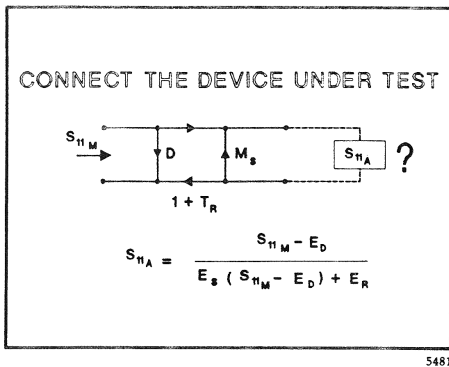


What about phase uncertainty? Phase uncertainty is a function of both  $s_{11}$  (the result after calibration) and  $\Delta s_{11}$ . The worst case phase error occurs when  $\Delta s_{11}$  is perpendicular to the value of  $s_{11}$ . Since  $\Delta s_{11}$  is a worst case value,  $\Delta\varphi$  which is defined as the arcsin of  $(\Delta s_{11}/s_{11})$  is the worst case phase uncertainty about  $\varphi_{11}$ .

3-11

$$\Delta S_{11} = S_{11M} - S_{11A} \cong D + T_R S_{11A} + M_S S_{11A}^2$$

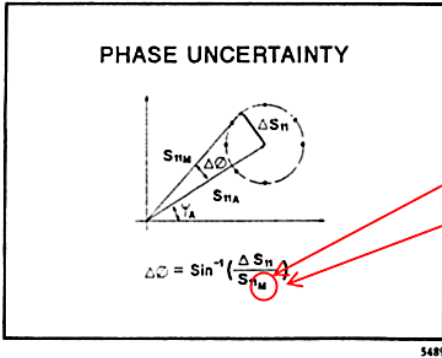
Fig. 4: Phase uncertainty  $\Delta\varphi$  and  $\Delta S_{11}$  [2]



(a) HP Seminar notes

(b) Author's

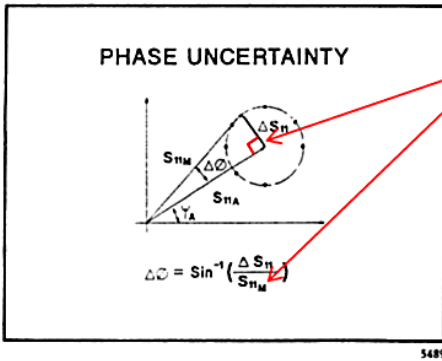
Fig. 5: One-port error model



What about phase uncertainty? Phase uncertainty is a function of both  $s_{11}$  (the result after calibration) and  $\Delta s_{11}$ . The worst case phase error occurs when  $\Delta s_{11}$  is perpendicular to the value of  $s_{11}$ . Since  $\Delta s_{11}$  is a worst case value,  $\Delta\phi$  which is defined as the arcsin of  $(\Delta s_{11}/s_{11})$  is the worst case phase uncertainty about  $\psi_{\dots}$

3-11

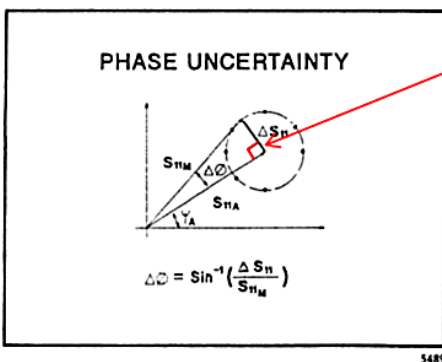
Fig. 6: Two (2) Inconsistencies



What about phase uncertainty? Phase uncertainty is a function of both  $s_{11}$  (the result after calibration) and  $\Delta s_{11}$ . The worst case phase error occurs when  $\Delta s_{11}$  is perpendicular to the value of  $s_{11}$ . Since  $\Delta s_{11}$  is a worst case value,  $\Delta\phi$  which is defined as the arcsin of  $(\Delta s_{11}/s_{11})$  is the worst case phase uncertainty about  $\psi_{\dots}$

3-11

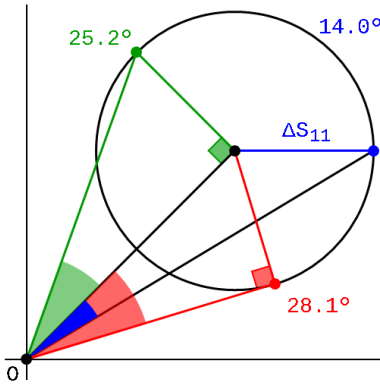
Fig. 7: One (1) Query



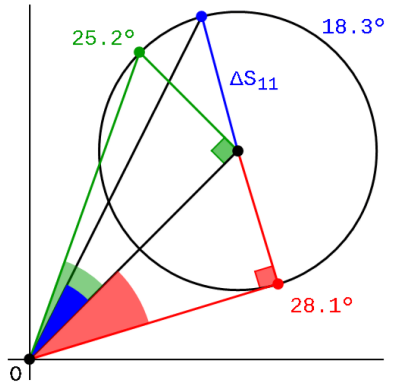
What about phase uncertainty? Phase uncertainty is a function of both  $s_{11}$  (the result after calibration) and  $\Delta s_{11}$ . The worst case phase error occurs when  $\Delta s_{11}$  is perpendicular to the value of  $s_{11}$ . Since  $\Delta s_{11}$  is a worst case value,  $\Delta\phi$  which is defined as the arcsin of  $(\Delta s_{11}/s_{11})$  is the worst case phase uncertainty about  $\psi_{\dots}$

3-11

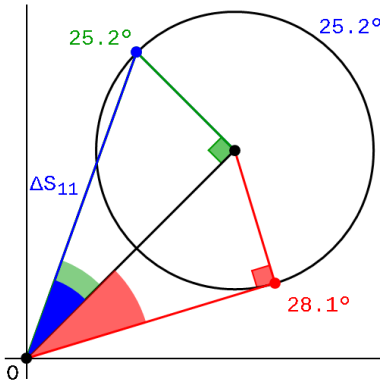
Fig. 8: One (1) Error



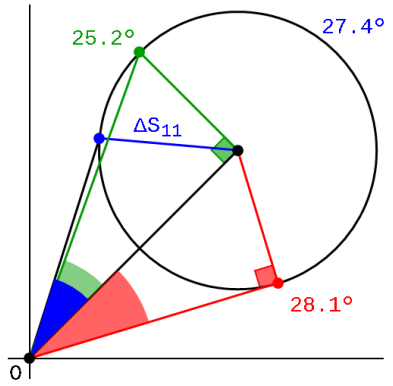
(a)



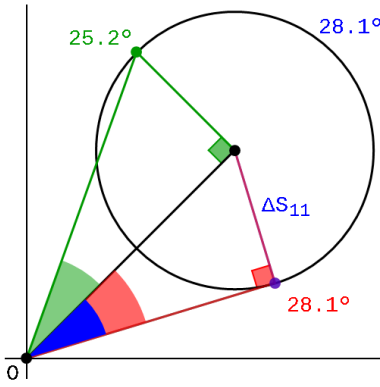
(b)



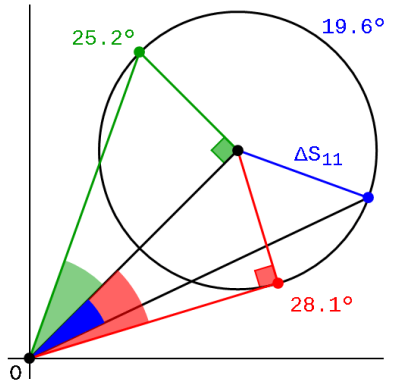
(c)



(d)



(e)



(f)

Fig. 9: Six characteristic frames for  $\Delta\phi$  values

2. Another issue has to do with the adopted interval of values for phase and its uncertainty. For example, in a presentation of 33rd ANAMET meeting [3], there is a figure showing the phase versus frequency covering the range of values  $[-5000^\circ, 1000^\circ]$ , while the following, right after that, calculation of electrical length requires the phase to be in radians.

In a presentation of 34th ANAMET meeting [4], an example exists for the reflection coefficient of Thru showing: (1) the linear magnitude with uncertainty greater or equal to 0 and omitting the negative values, (2) the phase is given in the interval  $(-\pi, \pi]$ , while (3) its uncertainty is in the interval  $(-2\pi, 2\pi]$ .

Finally, a presentation of the same ANAMET meeting, as and its corresponding previously published paper [5, 6], shows  $S_{11}$ ,  $S_{22}$  "reflection phase" for a 17mm air line in the range of  $-2500^\circ$  to  $0^\circ$ , which means: six (6) times around the circle plus  $340^\circ$ .

3. Since we basically have ratios and products of complex numbers  $S_{ij}$  to deal with, we are interested in difference and addition of phase angles. If we accept the  $(-\pi, \pi]$  interval from (1) for any phase angle, then obviously

the difference of two phase angles will be in the interval  $(-2\pi, 2\pi)$  and the addition will be in  $(-2\pi, 2\pi]$ . These intervals cover the unit circle for determining the phase more than once, thus destroying the "1-1" correspondence.

The shape of the correspondence for both difference and addition in the above mentioned intervals is shown in Fig. 10(a) in a Cartesian plot, where the principal interval is indicated by the thick black frame. Fig. 10(b), (c) shows its right and left extension respectively, where we have taken care to keep the same  $360^\circ$  range as in the principal interval. The values of angles that are not included are indicated with an open circle. It is clear that in each of these two intervals there is a discontinuity described by the shown jigsaw function. Moreover, the  $\phi$  principal angles result from different  $\phi_i$  value, for example quadrant I results from the interval  $(0^\circ, 90^\circ)$  and also from the interval  $(-360^\circ, -270^\circ)$ . In both figures the number of each quadrant is shown for both axes.

In order to correctly compute the complex number we discriminate two cases: (i) if  $\phi_i \in (-360^\circ, 360^\circ)$  then we use the relations:

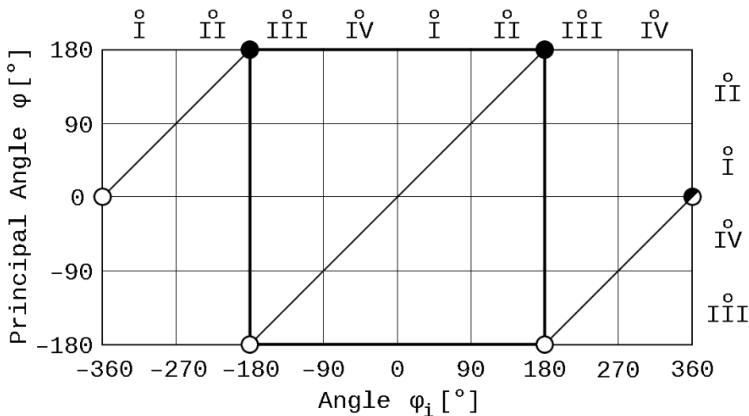
$$\varphi = \begin{cases} \varphi_i + 360^\circ, & \varphi_i \leq -180^\circ \\ \varphi_i - 360^\circ, & \varphi_i > +180^\circ \\ \varphi_i, & -180^\circ < \varphi_i \leq 180^\circ \end{cases} \quad (4)$$

and (ii) if the angle is outside the interval  $(-360^\circ, 360^\circ)$  then a two step procedure is needed: (ii.1) we ap-

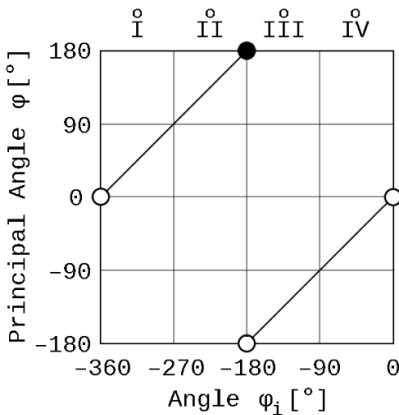
ply the well known Euclid's division lemma extended to negative dividend or negative divisor to find the signed remainder  $\varphi_i$  in  $(-360^\circ, 0^\circ)$ , or  $(0^\circ, 360^\circ)$  as shown in Fig. 10(b), (c) respectively,

$$\varphi' = k 360^\circ + \varphi_i, \quad k \in \mathbb{Z} \quad (5)$$

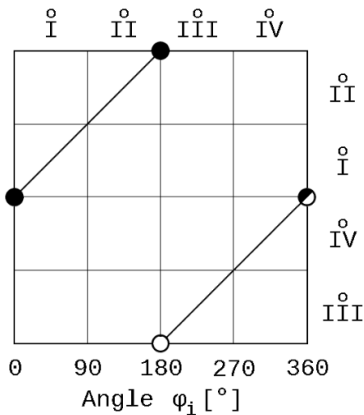
and (ii.2) apply (4) for  $\varphi_i$ .



(a) Jigsaw function



(b)  $\varphi_i \leq -180^\circ$



(c)  $+180^\circ < \varphi_i$

Fig. 10: Principal angle  $\varphi$  in terms of angle  $\varphi_i$

In order a) to reveal the problem, and b) to amplify our thesis regarding these issues, two extreme examples are presented in Fig. 11. The phase difference of blue point A with phase  $+177^\circ$ , in respect to  $-177^\circ$  phase of the blue point A' ( $\dot{z}_A/\dot{z}_{A'}$ ), is not  $354^\circ$  but  $-6^\circ$ , while the phase difference of red point B with phase  $-80^\circ$  in respect to  $+130^\circ$  phase of red point B' ( $\dot{z}_B/\dot{z}_{B'}$ ), is  $150^\circ$  and not  $-210^\circ$  as:

$$\Delta\varphi' = 177^\circ - (-177^\circ) = 354^\circ \Rightarrow$$

$$\Delta\varphi' > 180^\circ \Rightarrow \Delta\varphi = 354^\circ - 360^\circ \Rightarrow$$

$$\Delta\varphi = -6^\circ \quad \text{and}$$

$$\Delta\varphi' = -80^\circ - (+130^\circ) = -210^\circ \Rightarrow$$

$$\Delta\varphi' < -180^\circ \Rightarrow \Delta\varphi = -210^\circ + 360^\circ \Rightarrow$$

$$\Delta\varphi = 150^\circ$$

Finally, if we consider, as last example, the value  $-2400^\circ$  then from (5) we take:

$$\varphi' = -2400^\circ = -6 \times 360^\circ - 240^\circ$$

and thus from (4):

$$\varphi_i = -240^\circ < -180^\circ \Rightarrow$$

$$\varphi = -240^\circ + 360^\circ \Rightarrow \varphi = 120^\circ$$

Notably, there are various ways to use (4) and (5) in practice, because the program-

ming language in use may implement differently the functions integer and fractional parts of a real number.

It is important to keep always in mind what a Vector Network Analyzer, as our HP8505A, can measure and present as indications in [degrees]. Thus, for  $S_{ij}$  measurements the range for phase is  $\pm 180^\circ$  [7], as it corresponds to complex numbers, while this range may be different for measurements concerning the electrical length where also the final purpose is different, as occurred in the first ANAMET presentation above [3].

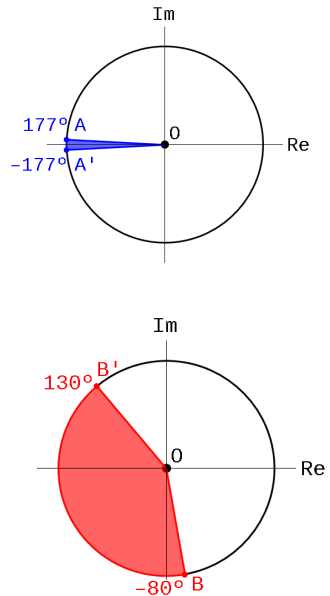


Fig. 11: Extreme  $\Delta\varphi$  examples



### On the Concepts of Magnitude and Phase Uncertainties

The full presentation of our exact estimation of VNA measurement uncertainties in comparison with the approximate ones by HP [2] begins with the contents of Tab. 1. This table contains the various expressions used for the magnitude and phase uncertainties under discussion. In the first row the  $\Delta S_{11}$ ,  $\Delta\phi$  in black print are the expressions produced by HP [2] and reproduced here in Fig. 4, where the used  $\approx$  symbol implies some undeclared there [2] sort of approximations. In the second row, the  $\Delta S_{11}^F$ ,  $\Delta\phi^F$  are the full -not approximated- expressions produced by us and resulting from the complex difference  $S_{11M}-S_{11A}$ , where in dark gray print are the additional terms, which are missing from the corresponding HP expressions [2], above. In the third row, the  $\Delta S_{11}^d$  and  $\Delta\phi^d$  in light gray print are correspondingly the absolute value of the difference between the complex numbers  $\hat{S}_{11M}$ ,  $\hat{S}_{11A}$  and the phase difference between them. Finally, in the fourth row in red print are shown the defined by us polar DEIs, differential error intervals, for the magnitude and phase of  $S_{11}$ .

Two related examples are

shown in Fig. 12 and Fig. 13: the first for a Box surround a 50 Ohm dc resistance, and the second for a UHF Ground Plane Antenna (GPA) [8]. Blue color is used for the Differential Error Region (DER) and green for the rectangular DEIs, real and imaginary part. The numeric evaluation of  $\Delta S_{11}$  results  $2^{(7 \times 2)} = 2^{14} = 16,384$  points from  $N = 7$  (7 complex variables) interval endpoints, as it was explained in detail in [8, 9].

Almost all of  $\Delta S_{11}$  points belongs to  $S_{11}$ -DER for the selected frequency frames for both these DUTs.  $\Delta S_{11}$  underestimates the systematic error for the first and overestimates the error for the second. All the corresponding values for magnitude and phase, and their uncertainty are given in Tab. 2, colored accordingly. For the Box it is obvious that the full expression of  $\Delta S_{11}$  does not give a different result, but for the antenna there is some difference. The DEIs are given in absolute value.

Two AVI videos were produced covering the measured frequency range: 1) [Box-DERS-DEIs.AVI] and 2) [GPA-DERS-DEIs.AVI]. Eight frames were selected for each DUT shown in Fig. 14 (28, 80, 301, 444, 600, 782, 990, 1211 MHz) and Fig. 15 (600, 652, 700, 800, 816, 856, 900, 1000 MHz) re-

spectively, with all the corresponding values with their colors above each frame.

For the 50-Ohm Box a notable case occur at the lowest frequency of 2 MHz where the reflection coefficient  $S_{11}$  is nearly 0. The same behavior is true for the next 4 frames, that is, for 15, 28, 41 and 54 MHz. In Fig. 14(a),

the results for 28 MHz are shown with the orange point to correspond to the origin 0 of the coordinate system.  $\Delta S_{11}$  is very small (black/grey color), while this is a special case for our DER which contains the origin 0 and it gives a circle for the polar DEIs, as it is already explained in [10, 11].

Tab. 1: Magnitude and phase uncertainty expressions

Magnitude	Phase
$\Delta S_{11} = S_{11M} - S_{11A} \cong D + T_R S_{11A} + M_S S_{11A}^2$	$\Delta\varphi = \sin^{-1} \frac{\Delta S_{11}}{S_{11A}}$
$\Delta S_{11}^F = S_{11M} - S_{11A} = \frac{D + T_R S_{11A} + M_S S_{11A}^2 - D M_S S_{11A}}{1 - M_S S_{11A}}$	$\Delta\varphi^F = \sin^{-1} \frac{\Delta S_{11}^F}{S_{11A}}$
$\Delta S_{11}^d =  \dot{S}_{11M} - \dot{S}_{11A} $	$\Delta\varphi^d = \hat{S}_{11M} - \hat{S}_{11A}$
DEI: $\Delta S_{11}^-, \Delta S_{11}^+$	DEI: $\Delta\varphi^-, \Delta\varphi^+$

Tab. 2: Figs. 10 - 11 Magnitude and phase uncertainty

	50-Ohm Box - 873 MHz	GP Antenna - 976 MHz
$S_{11}$	$0.359 \angle -60.27^\circ$	$0.593 \angle -108.39^\circ$
$ S_{11} $	$\pm 0.034, \pm 0.034, \pm 0.028,$ $(0.040, 0.037)$	$\pm 0.193, \pm 0.200, \pm 0.211,$ $(0.061, 0.055)$
$\angle S_{11}^\circ$	$\pm 5.45, \pm 5.47, \pm 4.52,$ $(7.66, 7.72)$	$\pm 18.97, \pm 19.72, \pm 20.52,$ $(6.57, 6.64)$

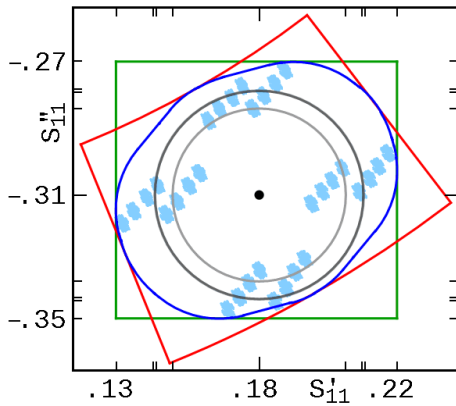


Fig. 12: 50-Ohm Box - 873 MHz

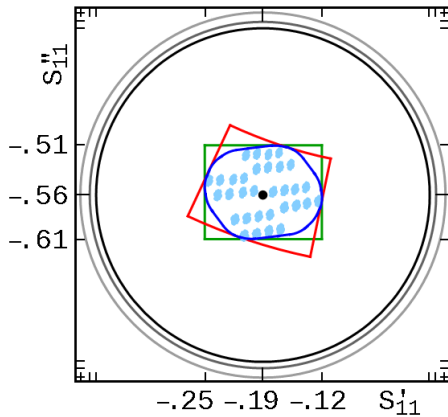


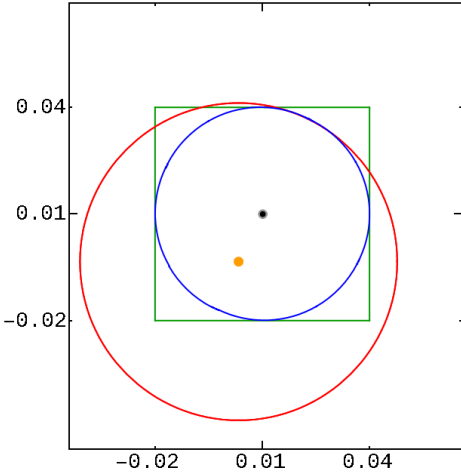
Fig. 13: GP Antenna - 976 MHz

For the antenna: The notable cases are for the frequencies 816, 820, 824 and 828 MHz. For example, at 816

MHz, shown in Fig. 15(e), the orange point of the origin 0 is inside the  $\Delta S_{11}$  circle and the arc sinus function cannot give an acceptable answer. At 828 MHz 0 is exactly on the periphery and still the arc sinus does not work. At 848 (Mag: 0.03, 0.03, 0.25, [0.03, 0.03], Phase: 14.1, 14.1, 142., [15.1, 15.1]) and 856 MHz (Mag: 0.04, 0.04, 0.27, [0.04, 0.03], Phase: 12.5, 12.6, 98.6, [11.9, 11.8]) shown in Fig. 13(f), the considered errors are comparable, and these are the only cases that this occurs.

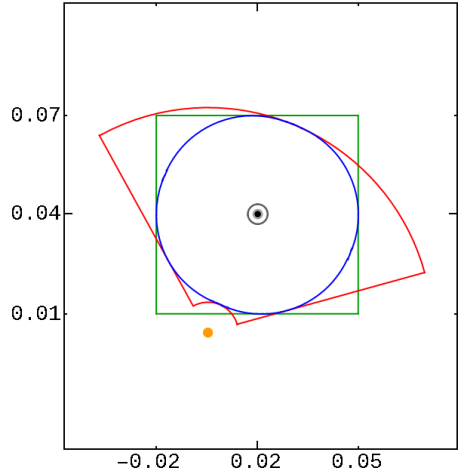
The  $\Delta S_{11}$  numeric evaluation in  $2^{(7 \times 2)} = 2^{14} = 16,384$  points, took  $\sim 5$  min on a Netbook Atom N270/1.6GHz, 1GB within Mathematica. The numeric evaluation of  $\Delta S_{21}$  for two-port measurements, demands:  $2^{(22 \times 2)} = \sim 18 \times 10^{12}$  points; at least:  $2^{(20 \times 2)} = \sim 1 \times 10^{12}$  points, if we exclude the Ex crosstalk system error [12]. So, we prepared the mini super computer of Fig. 16, an equivalent to 3x Cray-2 Super Computers, many years ago, with a 16 CPUs Cluster, 8x AMD Athlon X2/240 at 2.8 GHz, 16 GB RAM, in GNU/Linux 64-bit PelicanHPC 2.3.2 operating system [13], to try to evaluate all that points with parallel programming in a cluster.

Freq: 28MHz  
 Mag: 0.00 0.00 0.00 [ 0.03, 0.03]  
 Phase: 4.0 4.0 -2.0 [180.0, 180.0]



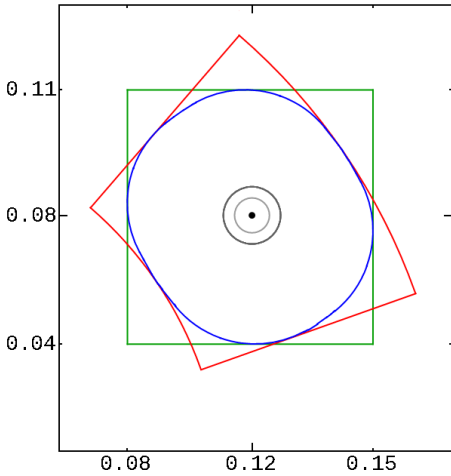
(a)

Freq: 80MHz  
 Mag: 0.00 0.00 0.00 [ 0.03, 0.03]  
 Phase: 4.4 4.4 -1.9 [ 51.7, 51.7]



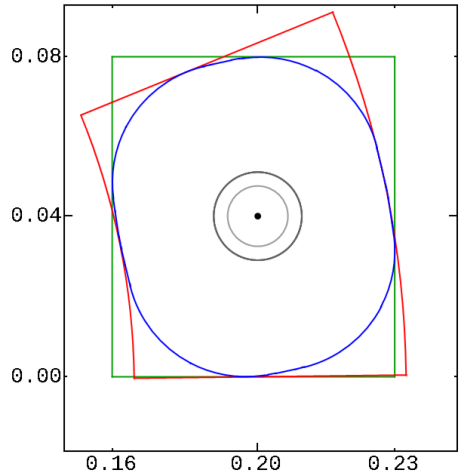
(b)

Freq: 301MHz  
 Mag: 0.01 0.01 0.00 [ 0.03, 0.03]  
 Phase: 3.3 3.3 -1.9 [ 14.9, 14.8]



(c)

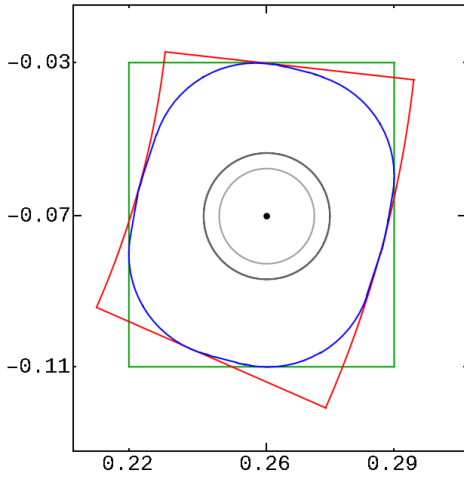
Freq: 444MHz  
 Mag: 0.01 0.01 0.01 [ 0.03, 0.03]  
 Phase: 2.9 2.9 -1.7 [10.8, 10.8]



(d)

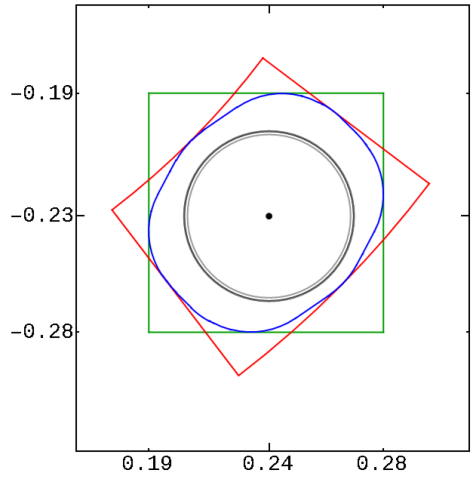
VNA UNCERTAINTY PART 7: REMARKS ON THE UNCERTAINTY NOTIONS

Freq: 600MHz  
 Mag: 0.02 0.02 0.01 [ 0.03, 0.03]  
 Phase: 3.6 3.6 -1.4 [ 8.6, 8.6 ]



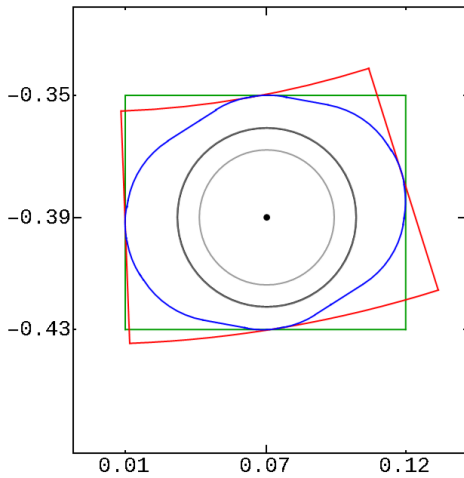
(e)

Freq: 782MHz  
 Mag: 0.03 0.03 0.03 [ 0.04, 0.04]  
 Phase: 5.4 5.4 2.5 [ 7.8, 7.8 ]



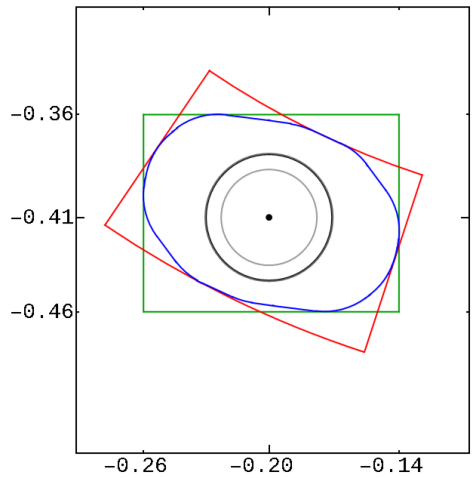
(f)

Freq: 990MHz  
 Mag: 0.03 0.03 0.03 [ 0.05, 0.04]  
 Phase: 4.8 4.8 0.6 [ 7.6, 7.7 ]



(g)

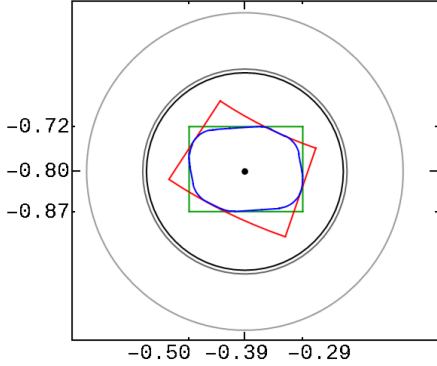
Freq: 1211MHz  
 Mag: 0.03 0.03 0.02 [ 0.05, 0.04]  
 Phase: 3.8 3.9 0.1 [ 8.0, 8.0 ]



(h)

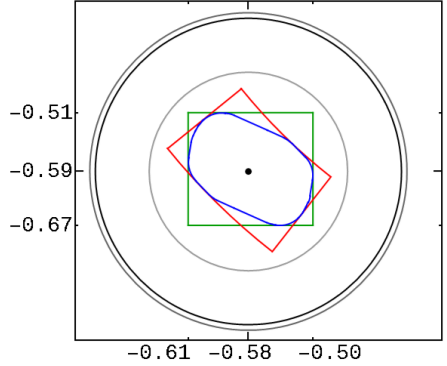
Fig. 14: Characteristic frames from [Box-DERS-DEIs.AVI]

Freq: 600MHz  
 Mag: 0.18 0.19 0.29 [ 0.09, 0.08 ]  
 Phase: 11.8 12.3 -18.8 [ 7.1, 7.1 ]



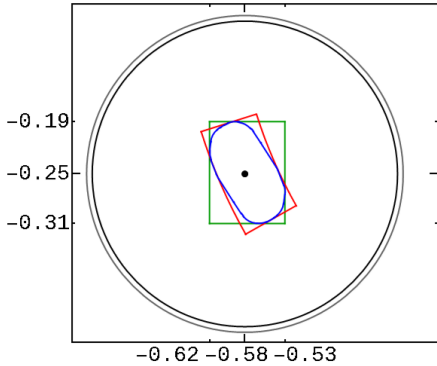
(a)

Freq: 652MHz  
 Mag: 0.21 0.22 0.14 [ 0.07, 0.06 ]  
 Phase: 14.6 15.1 -3.4 [ 6.4, 6.4 ]



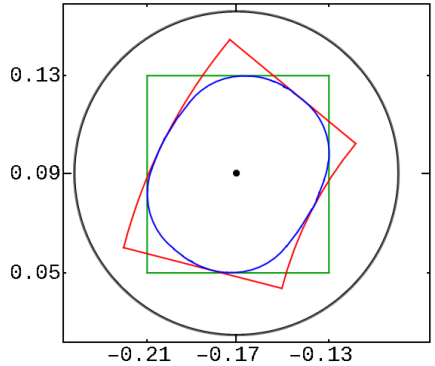
(b)

Freq: 700MHz  
 Mag: 0.19 0.20 0.39 [ 0.04, 0.03 ]  
 Phase: 17.3 18.0 38.5 [ 6.0, 5.9 ]



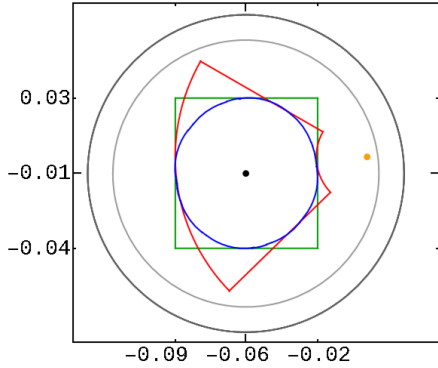
(c)

Freq: 800MHz  
 Mag: 0.07 0.07 0.27 [ 0.04, 0.03 ]  
 Phase: 20.7 20.8 95.1 [ 12.5, 12.6 ]



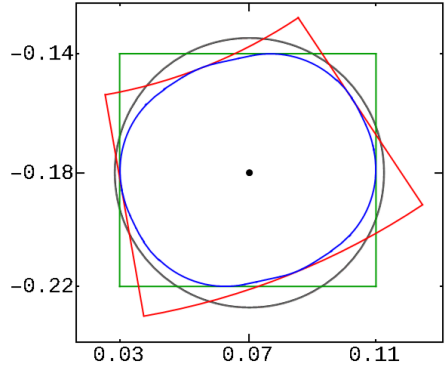
(d)

Freq: 816MHz  
 Mag: 0.07 0.07 0.06 [ 0.03, 0.03 ]  
 Phase: ----- -44.6 [ 37.5, 36.5 ]



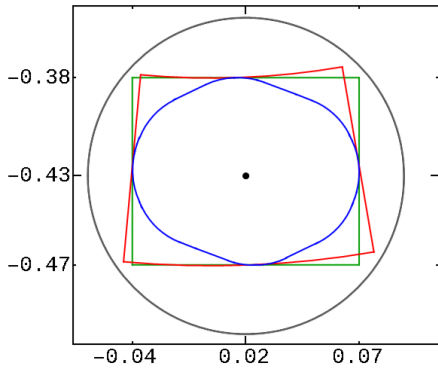
(e)

Freq: 856MHz  
 Mag: 0.04 0.04 0.27 [ 0.04, 0.03 ]  
 Phase: 12.5 12.6 98.6 [ 11.9, 11.8 ]



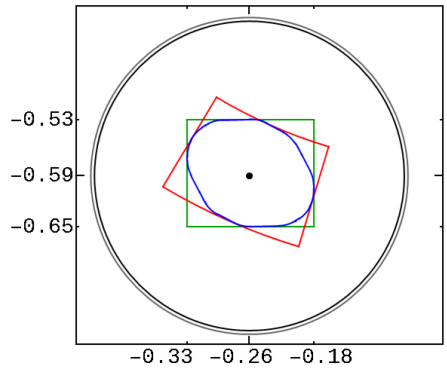
(f)

Freq: 900MHz  
 Mag: 0.08 0.08 0.79 [ 0.05, 0.04 ]  
 Phase: 10.4 10.4 161.0 [ 7.5, 7.4 ]



(g)

Freq: 1000MHz  
 Mag: 0.18 0.19 1.14 [ 0.06, 0.06 ]  
 Phase: 16.3 16.7 -147.0 [ 7.1, 7.0 ]



(h)

Fig. 15: Characteristic frames from [GPA-DERS-DEIs.AVI]

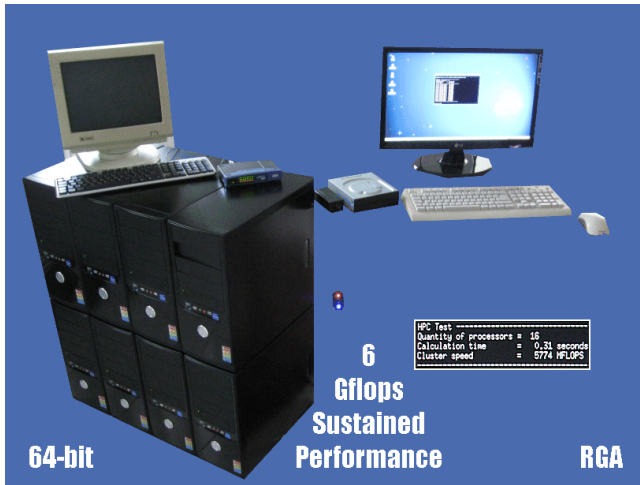


Fig. 16: The RGA mini-Super computer

### The Notion of Complex DER

In [14] someone can read for the S-parameters of a two-port error model: "Luckily you don't need to know these equations to use network analyzers". By the same reasoning, and probably to a greater extent, it is not necessary to know the expressions of the total differentials which produce the DERs' points. You need only the software tool in order to evaluate them [10, 11]. Some remarks for total differentials are:

1. Involve Linear relations by default.
2. Result in exact cyclo-polygonal regions in complex plane instead of approximate circles or ellipses.

3. Produce rectangular intervals, DEIs, for real and imaginary parts, and polar DEIs for magnitude and phase.

4. Demand careful use of mathematics.

5. Give answer to special cases.

6. Make possible the expression of interconnection between measurements and instrument specifications.

Another remarkable point for the advantages of total differentials came up during the PhD thesis [15] where a number of 3,059 possible antennas were presented, with 900+ figures, and 436 of them were constructed and measured. The production of three (3)



figures for only one (1) antenna is based on 42,432 measurements and almost 23 hours work with our HP 8505A were needed. And the query is: if a method based on statistical principles was used to determine the uncertainty for this specific antenna characteristics, then how many measurements would be necessary for the same antenna system in order to ensure a valid result? Statistics are simple enough, but time consuming under real conditions. A second query is: What errors do they really include?

**A Counter Example**

As a counter example to the full one-port calibration we bring back here the case of short response calibration for the reflection coefficient measurement of the UHF ground plane antenna [16]. The paper is open for permanent review at the given link in References below and for details. Fig. 17 shows the transformation of the one-port error model of Fig. 4(b), where the dashed boxes indicate the two system errors, directivity D and source match M, that do not exist.

Fig. 18 shows the comparison of the reflection coefficient after SLO and only S calibration with  $\rho$ -DERs and  $\rho_s$ -DERs as stripes with light and dark gray color respectively.

It is obvious that the two curves are very close, and that the DERs for  $\rho_s$  are much smaller. Fig. 19 shows the polar DEIs as stripes for the magnitude and phase of  $\rho$  and  $\rho_s$  where the  $\rho_s$  stripes are inside the corresponding  $\rho$  stripes almost in the whole frequency range.

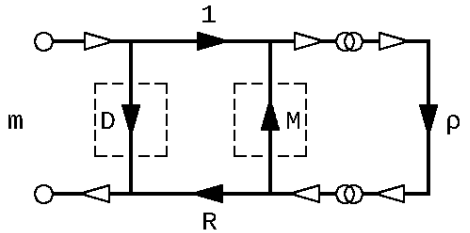


Fig. 17: One-port error model

Numerous results having as follows. Fig. 20 shows the two  $\rho$ -DERs after SLO and after S calibration for the selected frequency of 796 MHz, with their DEIs in rectangular and polar form. In Tab. 3 the values for magnitude, phase and their DEIs demonstrate the difference between the results of the two calibration techniques.

In short - response calibration only the R system error can be taken into account and it is given in Fig. 21 for the two calibration techniques in 3-D and 2-D. The black color is used for the full one-port SLO calibration and the red color for the S

short - response calibration. The corresponding R,  $R_s$ -DERs for the selected frequency are shown in Fig. 22.

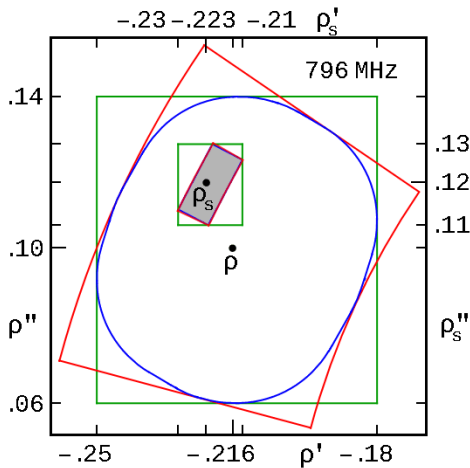


Fig. 20:  $\rho$ ,  $\rho_s$  DERs and DEIs

Tab. 4 contains the Min, Max and Mean value of the difference for magnitude and phase between the two cases. Obviously the R curve in Fig. 21 is the same but there is a phase difference.

In Fig. 23  $\rho$  and  $\rho_s$  are shown in 3-D with some selected DERs as beads for the two cases. The vertical axis corresponds to frequency from 600 to 1000 MHz. The figure was produced in Mathematica 7 and it was interactive. From this figure we create the film

[3D-DERs-DEIs.AVI] where some views are shown.

This is a counter example for the use of full one-port SLO calibration versus a simple response calibration with just one standard. The paper was uploaded to Agilent forum for open review back in 2011 with subject "Systematic Uncertainties in VNA Measurements" by pez, Total posts: 7, Total Views: 3,400+. We copied a comment which includes at least 5 interesting issues:

"... in older equipment and at low frequencies, where the directional couplers were well match (sometimes better than the loads you could purchase) but not applicable once you move away from the test port through any kind of test cable ...",

by D.J., PhD, Agilent Fellow, Total posts: 1,800+. The initial link of the forum is unfortunately not available any more, but the information has been archived by the authors at their site [17].

These issues need further investigation concerning both the older and the newer equipment for their ability to achieve a certain degree of satisfactorily accurate VNA measurements.

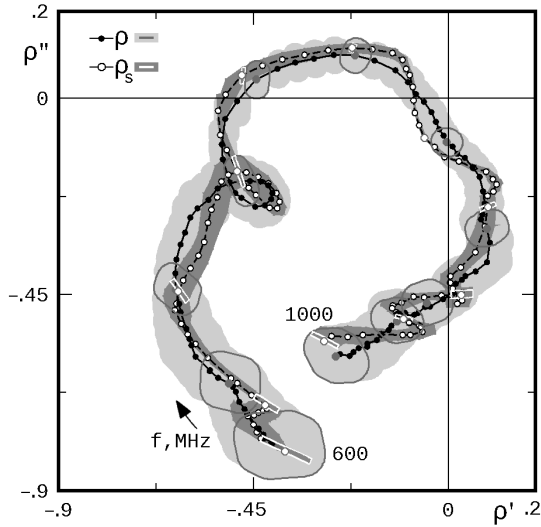


Fig. 18: Complex  $\rho$ -DERs and  $\rho_s$ -DERs in [600, 1000] MHz

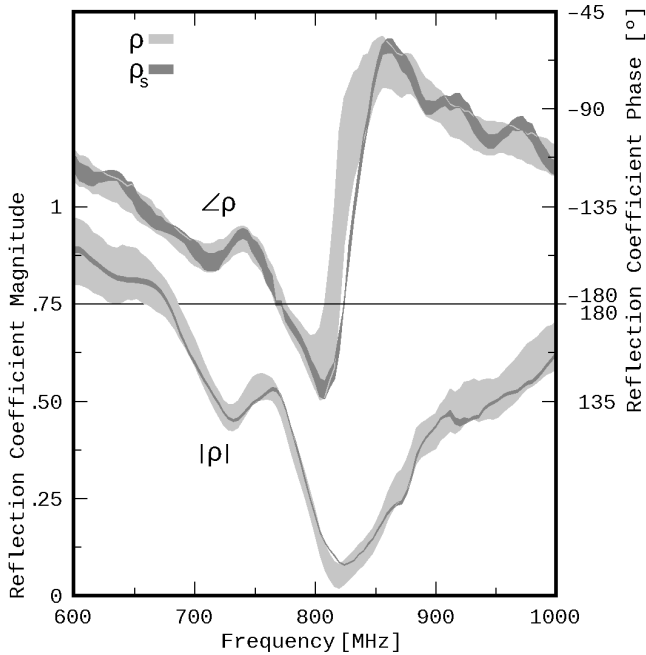


Fig. 19: Polar DEIs of reflection coefficient

Tab. 3:  $\rho$  and  $\rho_s$  polar DEIs

$\rho = 0.238 \angle 155.3^\circ$	
$ \rho $ (0.035, 0.033)	$\angle\rho$ (9.60°, 9.73°)
$\rho_s = 0.251 \angle 152.4^\circ$	
$ \rho_s $ (0.006, 0.003)	$\angle\rho_s$ (2.25°, 2.25°)

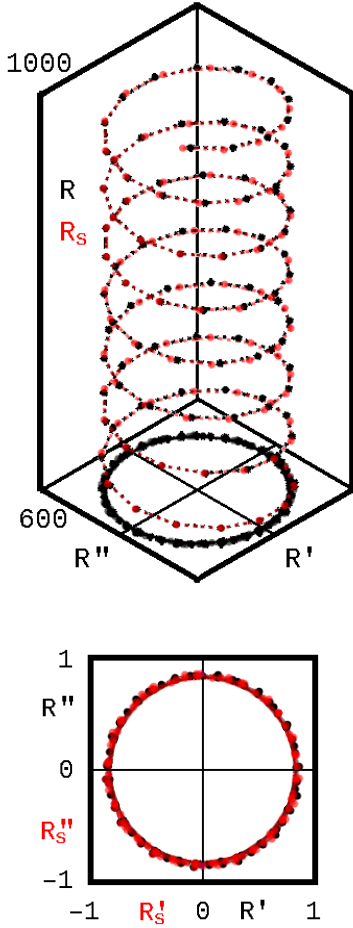


Fig. 21: R, Rs in 3-D and 2-D

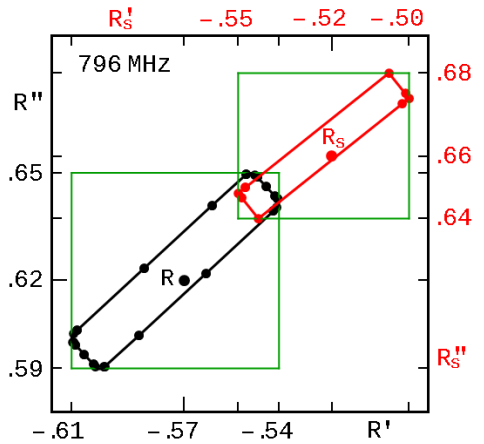


Fig. 22: R-DER, and Rs-DER

Tab. 4: R, Rs differences

600-1000 MHz	Min	Max	Mean
$ R  -  R_s $	$16 \times 10^{-6}$	$15 \times 10^{-3}$	$4.39 \times 10^{-3}$
$\angle R - \angle R_s$ [°]	0.04	10.61	4.65

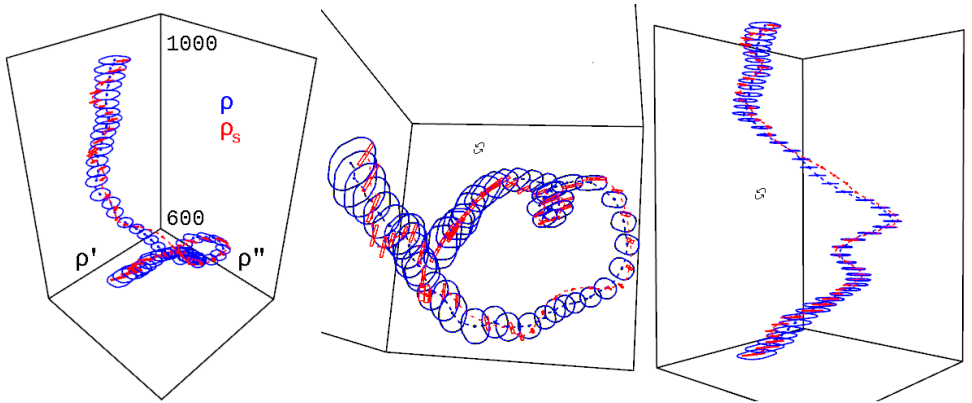


Fig. 23: Complex  $\rho$ ,  $\rho_s$  with selected DERs in [600, 1000] MHz

### Conclusion

Due to the one-to-one correspondence between the points of  $\mathbb{C}$  complex plane and  $\mathbb{R}^2$  often the complex numbers corresponding to ordered pairs of real numbers are treated as vectors. This is how the "Vector" designation has been established for the Network Analyzer which measures magnitude and phase. We have tried here to demonstrate the confusion caused by the meaning of the word "phase" at least in relation to measurements of  $S_{ij}$  in the frequency domain. We did not deal at all for example with the area of measurements with a VNA related to Electrical Length. We accept the use of the terms Magnitude and Phase imposed by VNA but emphasize that the pairs are in our opinion (Magnitude, Argument)

and (Amplitude, Phase), the latter coming closer to physical reality.

We have shown that a hasty and careless reading of the available literature can lead to misunderstandings, and we have compared different ways of calculating the uncertainty in  $S_{11}$  reflection coefficient measurement for two loads with the additional visual aid provided by videos. Through these, the variety of different frequency-dependent produced DERs was revealed. Additionally, their 3-D representation provides a complete picture of what one can expect for the behavior of the DUTs we measure, at least the open ones, under real-world conditions.

Some positive points for the advantageous use of our own method of DERs and DEIs

were mentioned and the size of the problem for a direct numerical calculation of  $\Delta S_{ij}$  was highlighted.

Finally, with a counter-example we showed that there is a case with a simple and fast Short-Response only cal-

ibration where it is possible to get the required accuracy for the  $S_{11}$  measurements although this conclusion resulted a-posteriori.

However, many issues still exist, as we stated, and require further study.

## References

- [1] HP 8505A RF Network Analyzer Basic Measurements, Application Note 219, Palo Alto California, November 1978, p. 3
- [2] HP, "Vector Measurements of High Frequency Networks", Hewlett-Packard, HP publication number 5958-0387, 1989, pp. 3-9, 3-11
- [3] Horibe M., "Verification of VNA reflection phase measurements", 33rd ANAMET meeting, Rohde & Schwarz, Fleet, 10 May 2010  
"[http://resource.npl.co.uk/docs/networks/anamet/members\\_only/meetings/33/20100511\\_anamet33\\_masahiro.pdf](http://resource.npl.co.uk/docs/networks/anamet/members_only/meetings/33/20100511_anamet33_masahiro.pdf)"
- [4] Salter M., "Uncertainty in magnitude and phase by analytical propagation of distributions", 34th ANAMET meeting, National Physical Laboratory, Teddington, 21 October 2010  
"[http://resource.npl.co.uk/docs/networks/anamet/members\\_only/meetings/34/20101021\\_anamet34\\_salter1.pdf](http://resource.npl.co.uk/docs/networks/anamet/members_only/meetings/34/20101021_anamet34_salter1.pdf)"
- [5] Hoffmann J.P, Leuchtman P, Ruefenacht J., Wong K., "S-parameters of slotted and slotless connectors", 34th ANAMET meeting, National Physical Laboratory, Teddington, 21 October 2010
- [6] Hoffmann J.P, Leuchtman P, Ruefenacht J., Wong K., "S-parameters of Slotted and Slotless Coaxial Connectors", 74th ARFTG Microwave Measurement Conference, 2009, DOI: 10.1109/ARFTG74.2009.5439109  
"<https://ieeexplore.ieee.org/document/5439109>"
- [7] HP 8505A Network Analyzer, High Performance RF Network Analyzer, Technical Data, January 1979, Palo Alto California, USA, pp. 14 - 17

- [8] Yannopoulou N., Zimourtopoulos P., "Total Differential Errors in One Port Network Analyzer Measurements with Application to Antenna Impedance", Radioengineering, Vol. 16, No. 2, June 2007, pp. 1 - 8  
"www.radioeng.cz/fulltexts/2007/07\_02\_01\_08.pdf"
- [9] Yannopoulou N.I., Zimourtopoulos P.E., "Measurement Uncertainty in Network Analyzers: Differential Error Analysis of Error Models Part 1: Full One-Port Calibration", FunkTechnikPlus # Journal, Issue 1, Year 1, 2013, pp. 17 - 22  
"www.ftpj.otoiser.org/issues/html/ftpj-issue-01-e4-lo4104-pdfc171-ia1.htm" (1-2)
- [10] Yannopoulou N.I., Zimourtopoulos P.E., "Measurement Uncertainty in Network Analyzers: Differential Error DE Analysis of Error Models Part 6: FLOSS - Software Tools", FunkTechnikPlus # Journal, Issue 26-27, Year 9, 2022, pp. 7 - 26  
"www.ftpj.otoiser.org/issues/html/ftpj-issue-26-27-lo4104-pdfc171-ia1.htm" (26-27-1)
- [11] Yannopoulou N.I., Zimourtopoulos P.I., "Complex Differential Error Regions: Software Tools", 31st ANAMET Club Meeting 02/04/2009  
"http://resource.npl.co.uk/docs/networks/anamet/members\_only/meetings/31/yannopoulou.pdf"  
Update 23/09/2019 "www.op4.eu/anamet/31"
- [12] Yannopoulou N.I., Zimourtopoulos P.I., "Measurement Uncertainty in Network Analyzers: Differential Error DE Analysis of Error Models Part 5: Step-by-Step Graphical Construction of Complex DE Regions and Real DE Intervals", FunkTechnikPlus # Journal, Issue 16, Year 5, 2018, pp. 7 - 25  
"www.ftpj.otoiser.org/issues/html/ftpj-issue-13-16-lo4104-pdfc171-ia1.htm" (13-16-1)
- [13] Creel M., PelicanHPC Tutorial, UF AE and IAE Working Papers from Unitat de Fonaments de l'Anàlisi Econòmica (UAB) and Institut d'Anàlisi Econòmica (CSIC)  
"https://econpapers.repec.org/paper/aubautbar/749.08.htm"
- [14] Ballo D., "Network Analyzer Basics", Hewlett-Packard Company, 1998, pp. 58

- [15] Yannopoulou N.I., "Study of monopole antennas over a multi-frequency decoupling cylinder", PhD Thesis, EECE, DUTH, February 2008 (in Hellenic), pp. (6-38) - (6-39) "[www.didaktorika.gr/eadd/handle/10442/20920?locale=en](http://www.didaktorika.gr/eadd/handle/10442/20920?locale=en)"
- [16] Yannopoulou N.I., Zimourtopoulos P.I., "Measurement Uncertainty in Network Analyzers: Differential Error Analysis of Error Models Part 3: Short One-Port Calibration - Comparison", FunkTechnikPlus # Journal, Issue 2, Year 1, 2013, pp. 41 - 49 "[www.ftpj.otoiser.org/issues/html/ftpj-issue-02-e4-lo4104-pdfc171-ia1.htm](http://www.ftpj.otoiser.org/issues/html/ftpj-issue-02-e4-lo4104-pdfc171-ia1.htm)" (2-3)
- [17] Agilent Discussion Forums, "[https://www.antennas.gr/anamet/35/Agilent\\_Discussion\\_Forums-View\\_topic-Systematic\\_Uncertainties\\_in\\_VNA\\_Measurements.htm](https://www.antennas.gr/anamet/35/Agilent_Discussion_Forums-View_topic-Systematic_Uncertainties_in_VNA_Measurements.htm)"
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\* The Constitution of Greece, Article 12(3) – 2008:

[www.hellenicparliament.gr/en/Vouli-ton-Ellinon/To-Politevma](http://www.hellenicparliament.gr/en/Vouli-ton-Ellinon/To-Politevma)

\* The Hellenic Supreme Court of Civil and Penal Law:

[www.areiospagos.gr/en](http://www.areiospagos.gr/en) – Court Rulings:Civil|A1|511|2008

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## FRONT COVER VIGNETTE

A faded synthesis of an anthemion rooted in a meandros

The thirteen-leaf is a symbol for a life tree leaf.  
"Herakles and Kerberos", ca. 530–500 BC,  
by Paseas, the Kerberos Painter,  
Museum of Fine Arts, Boston.

[www.mfa.org/collections/object/plate-153852](http://www.mfa.org/collections/object/plate-153852)

The simple meandros is a symbol for eternal immortality.  
"Warrior with a phiale", ca. 480–460 BC,  
by Berliner Maler,  
Museo Archeologico Regionale "Antonio Salinas" di Palermo.

[commons.wikimedia.org/wiki/  
File:Warrior\\_MAR\\_Palermo\\_NI2134.jpg](https://commons.wikimedia.org/wiki/File:Warrior_MAR_Palermo_NI2134.jpg)