Measurement Uncertainty in Network Analyzers: Differential Error Analysis of Error Models Part 1: Full One-Port Calibration

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Abstract

An analytical method was developed to estimate errors in quantities depended on full one-port vector network analyzer (VNA) measurements using differentials and a complex differential error region (DER) was defined. To evaluate the method, differences instead of differentials were placed over a DER which was then analyzed and compared with another commonly used estimated error. Two real differential error intervals (DEIs) were defined by the greatest lower and least upper bounds of DER projections. To demonstrate the method, a typical device under test (DUT) was built and tested against frequency. Practically, a DER and its DEIs are solely based on manufacturer's data for standard loads and their uncertainties, measured values and their inaccuracies.

Introduction

In full one-port measurements with a VNA of real characteristic impedance Z_0, a DUT with impedance Z has a reflection coefficient ρ defined by

$$\rho = (Z - Z_0)/(Z + Z_0)$$

and related to its measured value m by the bilinear transformation

$$\rho = (m - D)/[M(m - D) + R]$$

in terms of errors D, M and R [1]. This transformation can be uniquely determined from given distinct ρ_n, n = 1, 2, 3 and respectively known m_k, k = n [2].

Research

We considered ρ_n, m_k as the elements of given ordered triples (A, B, C), (a, b, c), solved the resulting system and appropriately expressed its solution by

$$F = \sum cC(B - A)$$

$$D = \sum abC(A - B)/F$$

$$M = \sum c(B - A)/F$$

$$R = [\prod (A - B)(a - b)]/F^2$$

where ∑ and ∏ produce two more terms from the one shown, by rotation of the ordered triple elements. These errors were then considered as depended on the independent variables ρ_n, m_k. Therefore, their differentials were expressed in the same manner by

$$dD = [\prod (a - b) \sum (B - C)BC\, dA + \sum (b - c)^2(B - A)(C - A) BC\, da]/F^2$$

$$dM = [\sum (a - b)(c - a)(B - C)^2dA - \prod (A - B) \sum (b - c)da]/F^2$$

$$dR = \{\sum [F + 2(a - b)B(A - C)] [(B - C)^2dA \prod (a - b) - (b - c)^2da \prod (A - B)]\}/F^3$$

After that, the differential of ρ was expressed by

$$d\rho = [-RdD - (m - D)^2dM - (m - D)dR + Rdm]/[M(m - D) + R]^2$$

and was considered depended, through dD, dM and dR, on $L = 7$ independent variables and their independent differentials: ρ_n, $n = 1, 2, 3$ and m_k, $k = n$ or $k = 0$ with $m_0 = m$.

The developed expressions were mechanically verified using a developed software program for symbolic computations.

Manufacturer's data for standard loads used in full-one port VNA measurements are substituted in ρ_n, and for their uncertainties in $d\rho_n$. Since Z_0 is real, the domain of each ρ_n is the closed unit circle [3]. For $|\rho_n| = 0$ or 1, care must be exercised to restrict its differential value onto its domain. The VNA measurements have specified bounded ranges for their modulus and argument, so that the domain of each m_k is a bounded circular annular with its center at the origin O of the complex plane. Measurement data are substituted in m_k and manufacturer's data for measurement inaccuracy in dm_k. Uncertainty and inaccuracy data outline domains for $d\rho_n$ and dm_k. If $z = |r|e^{j\varphi}$, stands for any of the independent variables and dz for its differential then the contribution of dz to dρ is a summation term of the form Wdz, with $W = |U|e^{jV}$, so that

$$Wdz = |U|e^{j(V + \varphi)}d|r| + |U|e^{j(V + \varphi + \pi/2)}|r|d\varphi$$

where W is in fact a known value of the respective partial derivative and d|r|, dφ are the independent real differentials of the complex dz in polar form. Each expression Wdz outlines a contour for a partial DER around O. If $z \neq 0$, the partial DER is a parallelogram with perpendicular sides d|r| and |r|dφ, stretched or contracted by |U| and rotated by $(V + \varphi)$ around O. If $z = \rho_n = 0$, the partial DER is a circle with radius |U|d|r|. Accordingly, a DER is the sum of either L parallelograms or $(L - 1)$ parallelograms and 1 circle. DER is then a convex set with contour either a polygonal line with 4L vertices at most, or a piecewise curve composed of 4(L – 1) line segments and 4(L – 1) circular arcs at most. The greatest lower and least upper differential error bounds are the end-points of DEIs for the real and imaginary parts of dρ and result from the projections of DER for ρ on the coordinate axes. These conclusions can be generalized for any other quantity directly or indirectly depended on all, some or just one of the above independent variables and their differentials. Thus, the quantity has an L-term DER, where $7 \geq L \geq 1$. For example, the impedance Z of a DUT has the 7-term DER:

$dZ = 2Z_0 d\rho/(1 - \rho)^2$

Results

All of the following data are specified by manufacturers of the parts for our measurement system. This system operates from 1 to 1300 MHz with 100 Hz PLL stability and consists of a type-N $Z_0 = 50\ \Omega$ network analyzer, a number of support instruments and a set of standard loads. The standards are: a short circuit A, a matching load B with reflection coefficient 0.029 and an open circuit C with reflection coefficient 0.99 and phase accuracy ±2°. In the absence of manufacturer's data for A we considered its uncertainty equal to that of C. So, the following values were substituted in the developed expressions:

$A = -1$, $0 \leq d|A| \leq 0.01$, $-180° \leq d\varphi_A \leq -178°$ or $178° \leq d\varphi_A \leq 180°$,

$B = 0$, $|dB| = 0.029$,

$C = 1$, $-0.01 \leq d|C| \leq 0$, $-2° \leq d\varphi_C \leq +2°$

The annular domain for m_k of VNA is specified from 0 to –70 db in modulus and ±180 degrees in argument. Measurements m_k result with a decimal floating point precision of 4 digits, for both modulus and argument. We consider the modulus and argument of dm_k equal to ±1/2 of the unit in the last place of the corresponding mantissa in modulus and argument of m_k Consequently, our system produces a DER, either for ρ or Z, as a sum of $(L - 1) = 6$ parallelograms and 1 circle, with a contour of $(4L + 4L) = 48$ vertices at most.

A suite of developed software applications: (i) controls the system and collects the data in terms of frequency using the IEEE-488 protocol, (ii) processes the collected data and computes the vertices of DER and the end-points of its DEIs (iii) sketches pictures of DER for ρ and Z in terms of the frequency steps and make a film using them as frames.

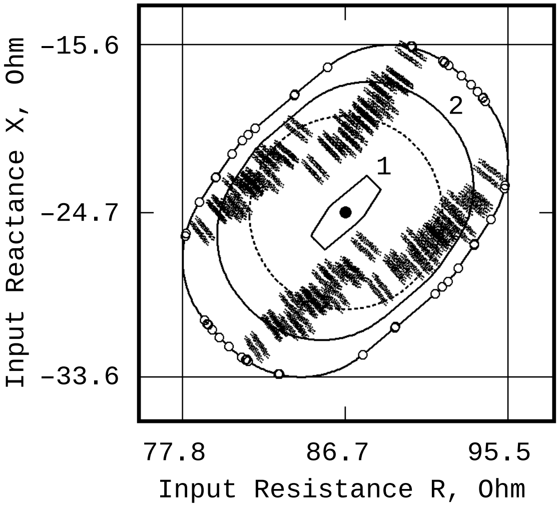

Fig. 1: A typical DER for the impedance Z

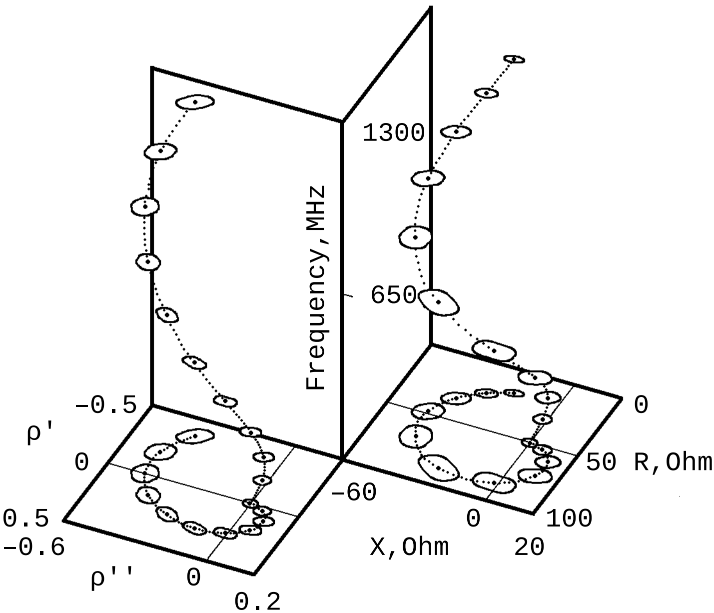

Fig. 2: DER for the reflection coefficient ρ and for its associated impedance Z against frequency

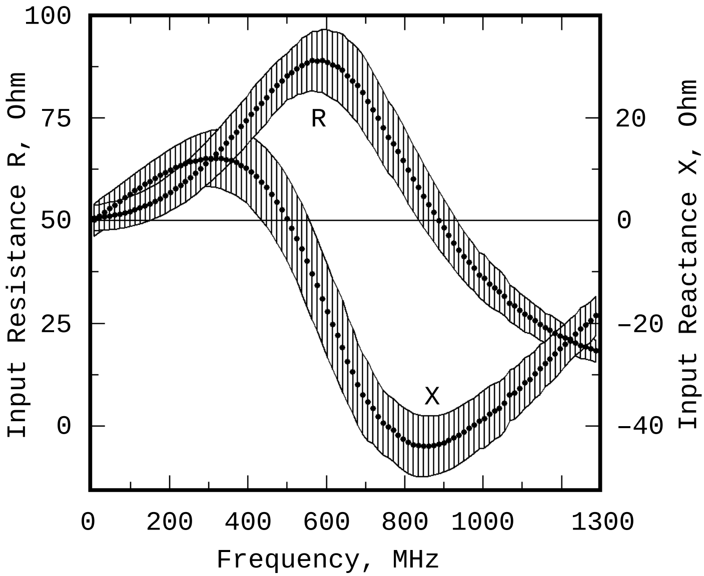

Fig. 3: Greatest lower and least upper differential error bounds for resistance R and reactance X against frequency

A typical resistor with a nominal DC impedance of 50 Ω ±20% tolerance was soldered on a type-N base connector and enclosed in an aluminium box to serve as a simple DUT for testing its Z from 2 to 1289 MHz in 13 MHz steps. The center frequency f_C = 639 MHz was chosen to reveal the details of the proposed method in Fig. 1, where the contour of a typical DER for Z is outlined with small circles as its vertices. This contour surrounds that of the 4-terms DER due to inaccuracy of measurements (1) and that of 3-terms DER for the uncertainty of loads (2). A properly circumscribed rectangle of DER shows graphically how the DEIs for R and X result. The commonly used error from the matching load only is shown as a dotted circle. This is in fact a 1-term DER which is surrounded from the contour of the DER by a factor of about 125% to 185% in all directions. Finally, in the same figure, 2^{7x2} differences ΔZ resulting from the same $d\rho_n$ and dm_k, dense enough to appear as stripes, were placed over DER to compare them with differential dZ values. Notably, almost all of ΔZ values are belong to DER while the computation time for these ΔZ exceeds that for DER by more than one order of magnitude. To demonstrate the method, a set of selected DER frames for ρ and Z are shown in Fig. 2, as beads on space curved filaments against frequency. Finally, the computed DEIs for R and X are shown in Fig. 3 against frequency.

Conclusion

The proposed method may be efficiently used in the same way, to successfully estimate errors in any quantity depended on full one-port vector network analyzer measurements.

References

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Preprint Versions

Differential Error Region of a Quantity Dependent on
Full One-Port Network Analyser Measurements
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arxiv.org/abs/physics/0612049

Follow-Up Research Paper

Total Differential Errors in One-Port Network Analyzer
Measurements with Application to Antenna Impedance
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