Near-field Method in Solving Inverse Scattering Problem of Spherical Electromagnetic Waves in Chiral media

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Abstract

We have developed two methods to study an inverse scattering problem of electromagnetic waves in chiral media, for a spherical scatterer perfect conductor. The first called Far-field inverse problem was described analytical in our previous work. The second called Near-field method is given in detail in the present paper. It is a geometrical method based on the scattered fields. Using Near-field experiments, in which the scattered field is measured at the source, we solve the corresponding inverse scattering problem that is to determine the coordinates of the center and the radius of the spherical scatterer.

Introduction

We study an inverse scattering problem of electromagnetic waves with time harmonic dependence applying the method of Near-field [1]-[3]. Spherical electromagnetic waves generated by a point source incident on a spherical scatterer perfect conductor in a chiral media.

Knowing the incident and scattered wave fields in the inverse problem we are looking for the coordinates of the sphere center and its radius.

The corresponding problem is solved by geometrical methods using either energy scattering cross-section, or the scattered fields. In the first case we refer to the inverse Far-field problem [4] and in the second case to a Near-field inverse problem.

The chiral materials exhibit the phenomenon of optical activity i.e., the phenomenon that in which the plane of polarizations of linearly polarized light is rotated as the light passes through an optically active medium.

The chirality is a property which is often found in nature and reflects the asymmetry in the spatial inversion. An electromagnetic wave into a chiral media is analyzed in a counter-clockwise

(LCP) and a clockwise (RCP) Beltrami field. Using vector spherical harmonic functions and appropriate expansions of the Beltrami fields and by extension the development of spherical electromagnetic waves in spherical wave functions, we calculate the exact solution of the scattering problem for a spherical perfect conductor in chiral media, and the corresponding back scattering [3]-[4].

In the inverse problem we calculate the scattered Near-field, when the source is in position $\mathbf{r} = \mathbf{r_0}$. A similar problem for achiral materials has been studied in the works [1], [2].

Statement of the problem

Consider a point source at the position $\mathbf{r}_0$ that produces spherical electromagnetic waves in a chiral media near a scatterer Ω^-, perfect conductor, i.e. the surface of the boundary condition is satisfied

$$\hat{\mathbf{n}} \times (\mathbf{E}^{inc}(\mathbf{r}) + \mathbf{E}^{sc}(\mathbf{r})) = \mathbf{0} \quad \tag{1}$$
$$r = a$$

We consider that the exterior $\Omega^+ = \mathbb{R}^3 / \Omega^-$ of the scatterer is homogeneous chiral media with fixed chirality β, dielectric constant ε and magnetic permeability μ.

A spherical incident electromagnetic wave $(\mathbf{E}^{inc}_{\mathbf{r}_0}(\mathbf{r})$, $\mathbf{H}^{inc}_{\mathbf{r}_0}(\mathbf{r}))$ with time harmonic dependence in accordance with the Bohren transformation, analyzed in spherical Beltrami fields $\mathbf{Q}^{inc}_{L,\mathbf{r}_0}(\mathbf{r})$ and $\mathbf{Q}^{inc}_{R,\mathbf{r}_0}(\mathbf{r})$ as follows:

$$\begin{cases} \mathbf{E}^{inc}_{\mathbf{r}_0}(\mathbf{r}) = \mathbf{Q}^{inc}_{L,\mathbf{r}_0}(\mathbf{r}) + \mathbf{Q}^{inc}_{R,\mathbf{r}_0}(\mathbf{r}) \\ \mathbf{H}^{inc}_{\mathbf{r}_0}(\mathbf{r}) = \dfrac{1}{i\eta}(\mathbf{Q}^{inc}_{L,\mathbf{r}_0}(\mathbf{r}) - \mathbf{Q}^{inc}_{R,\mathbf{r}_0}(\mathbf{r})) \end{cases} \tag{2}$$

where $\eta = (\mu/\varepsilon)^{1/2}$ is the intrinsic impedance of the chiral medium. The Beltrami fields satisfy the equations, [5], [6],

$$\begin{cases} \nabla \times \mathbf{Q}_{L,\mathbf{r}_0}(\mathbf{r}) = \gamma_L \mathbf{Q}_{L,\mathbf{r}_0}(\mathbf{r}) \\ \nabla \times \mathbf{Q}_{R,\mathbf{r}_0}(\mathbf{r}) = -\gamma_R \mathbf{Q}_{L,\mathbf{r}_0}(\mathbf{r}) \end{cases} \tag{3}$$

where γ_L, γ_R are wave numbers for Beltrami fields and are given by,

$$\gamma_L = \frac{k}{1 - k\beta}, \gamma_R = \frac{k}{1 + k\beta} \tag{4}$$

With $k = \omega(\varepsilon\mu)^{1/2}$, ω being the angular frequency. The indices L and R denote the LCP and RCP fields respectively. The spherical incident Beltrami fields with suitable normalization have the form, as defined in the following works: [4] (issue 2, p. 9, relations (4), (5)), and [3].

If $\mathbf{Q}^{inc}_{L,\mathbf{r}_0}(\mathbf{r})$ and $\mathbf{Q}^{inc}_{R,\mathbf{r}_0}(\mathbf{r})$ are Beltrami fields corresponding via transformation to Bohren in $\mathbf{E}^{inc}_{\mathbf{r}_0}$ and $\mathbf{H}^{inc}_{\mathbf{r}_0}$, then the scattering problem for the perfect conductor can be formulated in the following way: be found $\mathbf{Q}^{tot}_{L,\mathbf{r}_0}(\mathbf{r})$, $\mathbf{Q}^{tot}_{R,\mathbf{r}_0}(\mathbf{r})$, which belong to the space $C^1(\Omega^+) \cap C(\overline{\Omega^+})$, such that:

$$(\mathrm{i}): \begin{cases} \nabla \times \mathbf{Q}^{tot}_{L,\mathbf{r}_0}(\mathbf{r}) = \gamma_L \mathbf{Q}^{tot}_{L,\mathbf{r}_0}(\mathbf{r}) \\ \nabla \times \mathbf{Q}^{tot}_{R,\mathbf{r}_0}(\mathbf{r}) = -\gamma_R \mathbf{Q}^{tot}_{R,\mathbf{r}_0}(\mathbf{r}) \end{cases}$$

$$\mathbf{r} \in \Omega^+$$

$$(\mathrm{ii}): \hat{\mathbf{n}} \times \mathbf{Q}^{tot}_{L,\mathbf{r}_0}(\mathbf{r}) = -\hat{\mathbf{n}} \times \mathbf{Q}^{tot}_{R,\mathbf{r}_0}(\mathbf{r}) \qquad (5)$$

$$\mathbf{r} \in S = \partial\Omega^-$$

$$(\mathrm{iii}): \begin{cases} \hat{\mathbf{r}} \times \mathbf{Q}^{sc}_{L,\mathbf{r}_0}(\mathbf{r}) + \mathrm{i}\mathbf{Q}^{sc}_{L,\mathbf{r}_0}(\mathbf{r}) = o(\frac{1}{r}) \\ \hat{\mathbf{r}} \times \mathbf{Q}^{sc}_{R,\mathbf{r}_0}(\mathbf{r}) - \mathrm{i}\mathbf{Q}^{sc}_{R,\mathbf{r}_0}(\mathbf{r}) = o(\frac{1}{r}) \end{cases}$$

$$r \to \infty$$

The limits on radiation conditions (5(iii)), are taken uniformly in all directions $\hat{\mathbf{r}} \in S^2$, where S^2 is the unit sphere in $\mathbb{R}^3$ and $\hat{\mathbf{n}}$ is the outward normal unit vector perpendicular to the surface on the scatterer.

The incident electromagnetic wave $(\mathbf{E}^{inc}_{\mathbf{r}_0}(\mathbf{r}),\ \mathbf{H}^{inc}_{\mathbf{r}_0}(\mathbf{r}))$ on the scatterer Ω^- generates the corresponding scattered field $(\mathbf{E}^{sc}_{\mathbf{r}_0}(\mathbf{r}),\ \mathbf{H}^{sc}_{\mathbf{r}_0}(\mathbf{r}))$. The scattered electric field will be depended on the polarizations $\hat{\mathbf{p}}_L$, $\hat{\mathbf{p}}_R$, (see [4] (issue 2, p. 9,relation (6)) and will have the decomposition

$$\mathbf{E}^{sc}_{\mathbf{r}_0}(\mathbf{r} \mid \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) = \mathbf{Q}^{sc}_{L,\mathbf{r}_0}(\mathbf{r} \mid \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) + \mathbf{Q}^{sc}_{R,\mathbf{r}_0}(\mathbf{r} \mid \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) \qquad (6)$$

Where $\mathbf{Q}^{sc}_{L,\mathbf{r}_0}(\mathbf{r} \mid \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R)$ and $\mathbf{Q}^{sc}_{R,\mathbf{r}_0}(\mathbf{r} \mid \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R)$ are the corresponding scattered Beltrami fields which have the following behavior, when $r \to \infty$, [5], [7],

$$\mathbf{Q}^{sc}_{A,\mathbf{r}_0}(\mathbf{r} \mid \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) = h_0(\gamma_A r) \cdot \mathbf{g}_{A,\mathbf{r}_0}(\hat{\mathbf{r}} \mid \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) + O(\frac{1}{r^2}) \qquad (7)$$

with $A = L, R$. The functions $\mathbf{g}_{L,\mathbf{r}_0}$ and $\mathbf{g}_{R,\mathbf{r}_0}$ are the LCP and RCP far-field patterns respectively, which are defined by the following relationship [7],

$$\mathbf{g}_{A,\mathbf{r}_0}(\hat{\mathbf{r}} \mid \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) = \int_S \hat{\mathbf{n}} \times [\gamma_A \nabla \times \mathbf{E}^{sc}_{\mathbf{r}_0}(\mathbf{r}' \mid \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) - \varpi_A \gamma^2 \mathbf{E}^{sc}_{\mathbf{r}_0}(\mathbf{r}' \mid \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) e^{-\mathrm{i}\gamma_A \mathbf{r}\cdot\mathbf{r}'} ds(\mathbf{r}') \qquad (8)$$

With $\varpi_A = \begin{cases} -1, & A = L \\ 1, & A = R \end{cases}$, $\gamma^2 = \gamma_L \gamma_R$.

Near-field Inverse problem

Using the exact solution found in the paper [4] (issue 2, pp. 10-12) and considering appropriate asymptotic forms of the Bessel and Hankel functions in low frequencies, i.e. $|\gamma_A a| \ll 1$, calculate asymptotic expressions of the coefficients a_n^L, a_n^R, b_n^L and b_n^R. Specifically we have [3],

$$a_n^L \sim \frac{1+\beta k}{2\mathrm{i}\zeta_n^2(2n+1)}(\gamma_L a)^{2n+1}$$

$$\gamma_L a \to 0 \tag{9}$$

$$a_n^R \sim -\frac{\mathrm{i}}{2n\zeta_n^2}\frac{(1-\beta k)^{n+2}}{(1+\beta k)^{n+1}}(\gamma_L a)^{2n+1}$$

or

$$b_n^L \sim -\frac{\mathrm{i}}{2n\zeta_n^2}\frac{(1+\beta k)^{n+2}}{(1-\beta k)^{n+1}}(\gamma_R a)^{2n+1}$$

$$\gamma_R a \to 0 \tag{10}$$

$$b_n^R \sim -\frac{\mathrm{i}(1-\beta k)}{2\zeta_n^2(2n+1)}(\gamma_R a)^{2n+1}$$

Where

$\zeta_n = 1\cdot 3\cdot 5\cdots(2n-1) = (2n)!/(2^n n!)$

In the inverse problem we calculate the scattered Near-field in the source $\mathbf{r} = \mathbf{r_0}$. By the relation (21), in work [4], with $\mathbf{r} = \mathbf{r_0}$ and for LCP incidence, we obtain [3],

$$\mathbf{E}^{sc}_{\mathbf{r}_0}(\mathbf{r}_0 \,|\, \hat{\mathbf{p}}_L) =$$

$$= \sum_{n=1}^{\infty}\frac{n(n+1)}{2}B_n^L a_n^L\{(h_n(\gamma_L r_0)\hat{\mathbf{x}} + \tilde{h}_n(\gamma_L r_0)\hat{\boldsymbol{\psi}}) + \mathrm{i}(\tilde{h}_n(\gamma_L r_0)\hat{\mathbf{x}} - h_n(\gamma_L r_0)\hat{\boldsymbol{\psi}})\} + \tag{11}$$

$$+ \sum_{n=1}^{\infty}\frac{n(n+1)}{2}B_n^L a_n^R\{(h_n(\gamma_R r_0)\hat{\mathbf{x}} - \tilde{h}_n(\gamma_R r_0)\hat{\boldsymbol{\psi}}) + \mathrm{i}(-\tilde{h}_n(\gamma_R r_0)\hat{\mathbf{x}} - h_n(\gamma_R r_0)\hat{\boldsymbol{\psi}})\}$$

Using the asymptotic relations:

$$\frac{h_n(\gamma_A r_0)}{h_0(\gamma_A r_0)} \sim \frac{\zeta_n}{(\gamma_A r_0)^n}$$

$$\frac{\tilde{h}_n(\gamma_A r_0)}{h_0(\gamma_A r_0)} \sim \frac{-n\zeta_n}{(\gamma_A r_0)^{n+1}} \tag{12}$$

$$\tilde{h}_n(\gamma_A a) \sim -\frac{n\zeta_n}{\mathrm{i}(\gamma_A a)^{n+2}}$$

with $\gamma_A r_0 \to 0$, $A = L, R$ and the series,

$$\begin{cases} \displaystyle\sum_{n=1}^{\infty}\tau^{2n+1} = \frac{\tau^3}{1-\tau^2} \\ \displaystyle\sum_{n=1}^{\infty}n^2\tau^{2n+1} = \frac{\tau^3(1+\tau^2)}{(1-\tau^2)^3} \end{cases} \tag{13}$$

$$\sum_{n=1}^{\infty}(2n+1)\,\tau^{2n+1} = \frac{\tau(3\tau^2-\tau^4)}{(1-\tau^2)^2} \tag{14}$$

$$\sum_{n=1}^{\infty} n(2n+1)\tau^{2n+1} = \frac{\tau(3\tau^2+\tau^4)}{(1-\tau^2)^3} \quad (15)$$

$$\sum_{n=1}^{\infty} \frac{2n+1}{n}\tau^{2n+1} = \quad (16)$$

$$= \frac{\tau[\tau^2 \ln(1-\tau^2) + 2\tau^2 - \ln(1-\tau^2)]}{\tau^2 - 1}$$

and because of the relationships:

$$\frac{\gamma_R}{\gamma_L} = \frac{1-\beta k}{1+\beta k}, \quad 0<\tau<1,$$

where $\tau = a/r_0$, we obtain from (11) [3],

$$\mathbf{E}^{sc}_{\mathbf{r}_0}(\mathbf{r}_0 \mid \hat{\mathbf{p}}_L) \sim \sim \frac{(1+\beta k)}{2(\gamma_L a)^2} \frac{\tau^5}{(1-\tau^2)^3} \cdot \hat{\mathbf{p}}_L \quad (17)$$

Therefore the measure of the scattered field $\mathbf{E}^{sc}_{\mathbf{r}_0}(\mathbf{r}_0 \mid \hat{\mathbf{p}}_L)$ is

$$\left|\mathbf{E}^{sc}_{\mathbf{r}_0}(\mathbf{r}_0 \mid \hat{\mathbf{p}}_L)\right| \sim \frac{(1+\beta k)}{2(\gamma_L a)^2} \frac{\tau^5}{(1-\tau^2)^3} \quad (18)$$
$$\gamma_L a \to 0$$

Similarly for the RCP incidence, we have that

$$\left|\mathbf{E}^{sc}_{\mathbf{r}_0}(\mathbf{r}_0 \mid \hat{\mathbf{p}}_R)\right| \sim \frac{(1-\beta k)}{2(\gamma_R a)^2} \frac{\tau^5}{(1-\tau^2)^3} \quad (19)$$
$$\gamma_R a \to 0$$

Finally, based on the relationships (18) and (19), we conclude that

$$\left|\mathbf{E}^{sc}_{A,\mathbf{r}_0}(\mathbf{r}_0 \mid \hat{\mathbf{p}}_A)\right| \sim \sim \frac{(1-\varpi_A \beta k)}{2(\gamma_A a)^2} \frac{\tau^5}{(1-\tau^2)^3} \quad (20)$$
$$\gamma_A a \to 0, \quad A = L, R$$

Choose a Cartesian coordinate system Oxψz, and five point-source locations, namely $O(0,0,0)$, $A_1(l,0,0)$, $A_2(0,l,0)$, and $A_4(0,0,2l)$, which are at (unknown) distances r_0, r_1, r_2, r_3 and r_4, respectively from the sphere's center K. The parameter l is a chosen fixed length. The sizes of the resulting five measurements $\left|\mathbf{E}^{sc}_{A,\mathbf{r}_j}(\mathbf{r}_j \mid \hat{\mathbf{p}}_A)\right|$ are

$$M_j = \frac{2(\gamma_L \gamma_R) l}{(1-\varpi_A \beta k)} \left|\mathbf{E}^{sc}_{A,\mathbf{r}_j}(\mathbf{r}_j \mid \hat{\mathbf{p}}_A)\right| \quad (21)$$

and

$$\rho_j = \frac{r_j}{l}, \quad b = \sqrt{\frac{a}{l}} \quad (22)$$

Where $j = 0,1,2,3,4$. Therefore we have the following five measurements

$$M_j = \frac{b^5 \rho_j}{(\rho_j^2 - b^4)^3} \quad (23)$$

with $\rho_j > b^2 > 0$. Applying the law of cosines in triangle KOA_4 and using that

$$r_j = \rho_j l \quad (24)$$

we obtain

$$\rho_4 = 2 + 2\rho_3 - \rho_0 \qquad (25)$$

Because of (24), we take

$$\rho_j^6 - 3\rho_j^4 b^4 + 3\rho_j^2 b^8 - - b^{12} - b^5 \frac{\rho_j}{M_j} = 0 \qquad (26)$$

Thus in equation (26) identified the ρ, with $j = 0, 1, 2, 3, 4$. If the radius a of the sphere is known, solve the system of six algebraic equations (25) and (26). Also if you consider that the radius a is too small so that $a \ll l$, then the relationship (23) shows [3],

$$M_j = \frac{b^5}{\rho_j^5} \qquad (27)$$

so we have the following system of equations

$$\begin{cases} \rho_4 = 2 + 2\rho_3 - \rho_0 \\ \rho_j^5 M_j - b^5 = 0 \end{cases} \qquad (28)$$

where $j = 0, 1, 2, 3, 4$.

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